

Goedel's Incompleteness Theorem

1. Cantor's Decimal Counting

It is best to get into this gently by going back to Cantor's work on the countability of the decimals, a problem that fascinated me when I first met it around 1954. Indeed, this topic seems an appropriate beginning to the entire enterprise that is this book, as well as being deeply strategic. Cantor's solution to the problem is a manageable little piece of thinking that you can get some control of before going on, something which is not true of the first page of chapter one of *Insight*. There Lonergan makes the point, in the first paragraph, of tackling something small, but the Archimedean problem is not so small: more than one expert has passed over it on the way to the second page.¹ What I wish to encourage in this chapter is a sufficient circling round this problem of Cantor to give a decent foothold on the slippery iceberg of Lonergan's view of the logic of being.²

¹My own treatment of it is available in Cantower 27. Available? Not existentially, no more than the present chapter makes Goedel's theorem available. My treatment of Archimedes' problem was the adventure of repeating it through six months, a few years ago, with different people - the age range was 8 to 80 - rising myself to fresh cherishing of the oddity of water supporting water in a glass poised among the galaxies. But always I began with a thin coat-hanger and two dangling balanced objects: bananas, whatever. What happens when you sink one banana in a glass of water? There is endless joy in repeating this, and that joy is related to the transition to a third meaning of generalized empirical method that is to be a topic in section 5 of this chapter.

²In the Preface I indicated that the direction of the book changed after the fourth section here, even the title. I decided that I should leave those four sections stand, and here the reference is to the original title of the book: *Lonergan's Transposition of the Logic of Being*, a title which still remains a valid statement of the drive of the book. As you shall see, and noted from the Preface, the shift gives a new emphasis to "out-referencing" such as occurred in the first footnote here. Following up such references changes the reading from a type of *haute vulgarization* to a serious foundational climb. More on this as we go along. This note, obviously, has been added to the original text of sections 1 - 4 of this chapter, and it seems best to continue to add such notes that are in line the new push. But I shall put such additions in square brackets so as to highlight the difficult shift.

It is interesting to note that we are heading into something that is pre-Lonergan in various senses. The Cantor problem was pre-Lonergan for me as a puzzle and indeed part of my stubborn battle with it appears below: one of my efforts to get round the Cantor proof - can you see how silly it is?³ But you might also think of what we venture into here, indeed in this entire chapter, as a pre-Lonergan venture, as a sort of push in history that parallels the push that I discussed in the first chapter of *Method in Theology and Botany*, where the push was the push towards the division of labour that is specified in *Method*. So, if you like, you can view this chapter as a strange new push towards *A Study of Human Understanding*. But let us venture into our topic, using the text that I wrote over thirty years ago. With the fresh hint about history's push you and I can both find that it reads differently now, and the drive of this little book is to transpose that reading towards being a reading within a flexible field,⁴ a reading towards a larger logic of being. But let us move into the text.

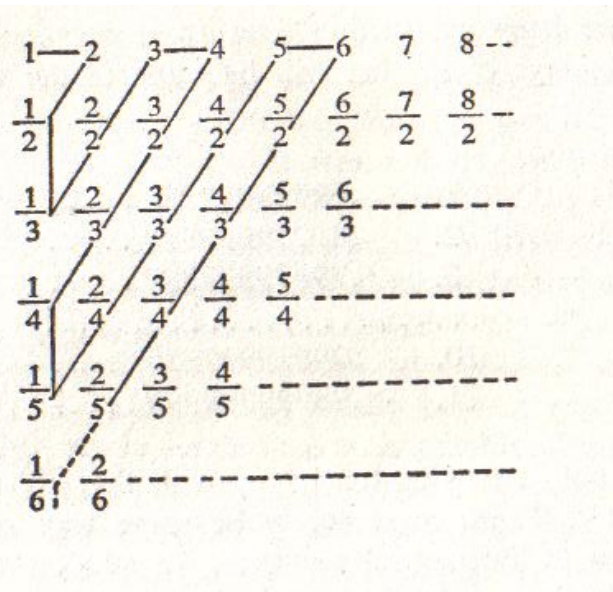
“You are perhaps beginning to appreciate the flexibility and variability of insight, the manner in which intelligence can neglect just what is to be neglected and attend to what is relevant. No room here for some simpleminded view of abstraction: the modern world is a world of calculus, surds, limitation theorems in mathematical logic, psychic illness, and political unrest. Intelligence can comfortably do mathematics with a green pen and have no difficulty in excluding green from the definition. But the procedure can be trickier, and so, a long scientific education may be needed to control the spontaneous anticipation of understanding. If the fractions between 0 and 1 can be put in one to one correspondence with the natural numbers, then surely the

³See note 5 below, and the text there.

⁴Unlike Lonergan in *Insight*, I am not writing this from a moving viewpoint - anyway, he regularly had to cheat! I am writing it more as “cajoling or forcing attention” (*Insight*, 398[423]), inviting you to pause, to deviate, to be amused and annoyed. Here I would wish you intrigued. What do I mean by *field*? You might try the index to **PL**, or venture into my reflections of *Lack in the Beingstalk*, chapter 3. [An additional reflection here regarding the new seriousness of the reference-following. In a serious science, and in a later stage of hedics, there would be much less need for such a strategy. One stays up-to-date with foundational shifts. Notice that our strange push, in the new context, is not just towards a freshened reading of *Insight*, but towards exercises that help towards an envisagement of the operation of The Standard Model in a century's time. In section 5, below, we pause over that challenge.]

decimals between 0 and 1 can be put in a similar correspondence? Indeed one might be inclined to say that the decimals might be easier to handle than the fractions. The problem here is to label the fractions - or decimals - so that for any given number the label can be produced, and vice versa. I might delay here to give hints, like I did in other examples, as to how to label the fractions. However, I go on immediately to give a handy way of labeling all the positive rational numbers. The reader, of course, could profitably delay here and try for himself or herself.

The trick is to lay out the rational numbers in a suitable array and then label them diagonal-wise as indicated in the diagram (there is a minor problem of leaving out the repeats). Now, you may name a rational number, I can find its label; you name a label, I'll point out its number.



Can we
decimals? The
would seem to be the following:

1. .0000000000.....
2. .1000000000.....
3. .2000000000.....
4. .3000000000.....

etc etc.

do the same for the
simplest labeling

Still, a little reflection reveals flaws In this list (does it?). Let's try another strategy: what we might call the mirror image method in that we subtract ones from the decimal "reversed". The beginnings of the list will help the reader to grasp what I mean:

1. .9999999999.....
2. .8999999999.....
3. .7999999999.....
-
10. .0999999999....
11. .9899999999.....

We may not in fact be too happy with that either.⁵ But we need not despair. There must surely be some way of lining up the decimals with the natural numbers. To admit that there isn't or couldn't be is to admit a certain lunacy into mathematics: as if there were more numbers than could be counted.

Let us assume that we have a suitable list, something like the last list suggested. Now, make a new decimal by taking the first number of the first decimal, the second from the second decimal, and so on. Change each of these by, say, one. (The acute reader will note a problem here). Now we have a new decimal which differs from all those on our supposed complete list."⁶

This is a piece of an introductory text which I used in classes during the twenty years 1974-94. The difficulty always was to add thinking, and thinking about self, to the standard form of students' reading. Do we still have that difficulty, you and I here now, or do you smile at similar memories of this struggle to turn people from reading about self-attention to actually doing it? Yet even if you share such memories, our efforts here are towards moving further into this context of reading. The moving eventually will be the cyclic moving described in *Method in*

⁵I referred earlier to this attempt of mine in 1954, when I was doing undergraduate mathematics: I thought it quite good at the time, for a brief Archimedean time!

⁶I am quoting from pages 25-6 of *Wealth of Self and Wealth of Nations. Self-Axis of the Great Ascent*, a book now available, free, on Website www.philipmcshane.ca Lonergan presents the same problem in a Goedelian context on pages 52-3 of *Phenomenology and Logic*.

Theology and Botany, and the context will be the shared sub-systematics freshened in that cycle through the seventh functional specialty.⁷ But we must move towards a grasp of that fuller context of contexts step by slow-step, where steps must be paused on in order to be leaped from.⁸

A first step is to return to the text quoted in order to do a re-think. Return? Can you return to it without turning to it? What is your control of the meaning of it? These can be discomfoting questions. Consider them a very rough form of linguistic feedback.⁹

So we ask now, in whatever fullness of perspective we can muster,¹⁰ What was Cantor doing, what are we doing, in the production of this reflection on the decimals? The mention of fullness of perspective places our efforts in quite a different context to the efforts demanded by Lonergan when he wrote *Insight*. Here I am moving more in the style of Archimedes' famous treatise *On Floating Bodies*.¹¹ Or, if you like, with something of the demanding assumptions that

⁷[I realize that this is already too complex a pointing, and so many other possibilities of complexification groan to get from my neuromolecules to the page. Perhaps you can recall that diagram from the end of *Lack in the Beingstalk*, repeated from *The Shaping of the Foundations*, a vortex of words twirling in and round towards *Mystery* at the centre? "It is only in the Eye of the Storm that one can name the Mystery". That diagram leaps into new meaning for you and for me as we move along here. But it can only be said in the dispersedness of slow narrative. There is a sense in which this entire chapter is really about introducing you self-referentially to the eye that is the I-surface of your minding.]

⁸[Again, the mood of the previous note leads me to twist an old phrase, steps "staring you, stairing you, in the face". The stairs stared at non-Platonically will mesh with the layers of languages of Hao Wang, mentioned in **PL**. See note 71 below.]

⁹The broad drive of linguistic feedback is mentioned in *Method in Theology* 88, note 32.

¹⁰[I would wish you not to be discouraged here. Muster whatever background you have without digressing into old or new texts in mathematics that you could make available. The task is a twisting towards a fresh discovery of you and eye - yes, that's the eye, reading now: shut one, to make it singular! - as you are. If you can muster the intellectual conversion briefly described in chapter 10 below, then are far out towards the lunacy of The Standard Model, psychically tuned to a community of the end of this century.]

¹¹See note 1 above. [The mix here, you will find, is odd. Archimedes is brutally axiomatic. Below you find some turns towards such obscurity, so dimensions of humour and play have to be added. By such means, self-digestively, we find out, grind out, who we are. This grinding was quite beyond Cantor and Goedel.]

characterize such a journal as *Journal of Symbolic Logic*.¹² That demanding assumption is to be the character of a mature systematic cycling in the later working of Hodics, and the book aims indeed at generating a support for that character: or, to make the point, I should say characters.¹³

What Cantor was doing, what we are doing, then, is attending to the mediating content of our shared inner word of mathematics. Why am I twisting back and forth between Cantor and ourselves, indeed between Cantor, me and you? Because we are edging towards a massive cultural shift of context, where *context* has the meaning of inner cultural word of the genuine thinking, mediating, towering, minority. A help here is to think of Lonergan's reflections on the adequate historian of mathematics.¹⁴ Such an adequate historian, up-to-date in his or her triply-luminous grasp of the field, has a context of sequentially linked systems controlling the reach of the question concerning Cantor or self.¹⁵

Note here the presence, within my context but perhaps not within yours, of the tone of a cyclic dynamic of hodics: the cycle aims at an improved view of history within systems, a view refined by the cycle through foundations to systematics. It is that refined systematics that informs the question of hodic interpretation, What is Cantor doing? Since Cantor, to a large extent, did not know what he was doing, you may think your way - but later! - to a larger respect both for contra-factual history's place in hodics, and for Lonergan's curious view of dialectic revealing the

¹²I give a more detailed description of the flawed dynamics of that journal in chapter 5 below.

¹³It is worth bearing in mind the beginning of the *Magna Moralia*, attributed to Aristotle: 'Since our purpose is to speak about matters to do with character, we must first inquire of what character is a branch. To speak concisely, then, it would seem to be a branch of nothing else than statecraft. For it is not possible to act at all in affairs of state unless one is of a certain kind, to wit, good. Now to be good is to possess the excellences. If therefore one is to act successfully in affairs of state, one must be of good character. The treatment of character then is, as it seems, a branch and starting point of statecraft'.

¹⁴The passage is quoted at note 7 in chapter 13 below.

¹⁵If you are unfamiliar with this notion of genetic structure you might take time now to venture more fully into the sketchings of chapters 7 and 13 below.

past as better than it was.¹⁶ There is that old Damon Runyon quote ” How are you doing? I’m doing what I can,”¹⁷ but now we seem to have the odd answer, “I’m doing better than I can.”

Cantor certainly, did not know what he was doing if we hold ourselves in the context of what Lonergan calls the third order of consciousness. “Distinct and concomitant, consciousness and intentionality are linked by two bridges of continuity and introspection. There is the bridge of continuity between the conscious human subject and the body in which he is incarnate: consciously her may move his fingers, hands, arms; intentionally, he may watch his moving fingers, hands, arms. There is also the bridge of introspection: it is a shift of attention by which we advert to the bridge of consciousness. Such adverting is both conscious and intentional, but it is of a second order, for it supervenes upon a prior consciousness and intentionality. Second-order consciousness is the presence of the subject to himself as introspecting: second order intentionality has a second-order object that in a first order is not an object but a datum of consciousness. Similarly, when as at present one introspects introspection, then there is a third-order consciousness and a third-order intentionality. There is a third-order consciousness, for consciously we advert to our adverting to our operations. There is a third-order intentionality whose object was, in the second order, not an object but the datum of consciousness that is the introspection being introspected..”¹⁸ That third order of consciousness, in its maturity, is to be

¹⁶*Method in Theology*, 251.

¹⁷*Insight*, 228[253].

¹⁸I am quoting, with corrections of typos, from pages 7-8 of a sketch of a chapter titled Method which I found as I was cataloging Lonergan’s Roman papers during 1974. The typescript continues I that file - which I labeled Batch V.7, a title retained I think in a later filing - up to page 9, but there is also a four-page handwritten sketch of a full chapter. The Batch is from February 1965, and contains also the “Discovery Pages” of functional specialization. Elsewhere I found what would seem to be in continuity with this tentative first chapter of a book: a typescript numbered pp. 8-23 in a file labeled A697. On page 14 of that typescript there is a Hegelian sweep that fits in with my effort to point to a massive communal global project, so I add here Lonergan’s reflection. “As the labor of introspection proceeds, one stumbles upon Hegel’s insight that the full objectification of the human spirit is the history of the human race. It is in the sum of the products of common sense and common nonsense, of the sciences and the philosophies, of moralities and religions, of social orders and cultural achievements, that is there is mediated, set before us in mirror in which we can behold, the originating principle of human

self-luminous control of total history: a distant fantasy at present. But Cantor was, in a twilight of the luminous darkness of circumstances,¹⁹ moving history right along.²⁰ Hodics is to be an ever-fresh recycling of this luminous global darkness of the circumstances of minding. Our present focus is on the telling of a telling little instance of the surge of energy's finality.

So, what are we at in our decimal struggle?²¹ Spontaneity and fingers has led us to the decimal system, but only an eccentricity can push us to ask about their ordered countability. Our spontaneity clamors for order, regularly bogged down in what we consider was already a decent order.²² As a young man, that clamor stood me against the diagonal proof of disorder. How goes it with you?

Is the eccentricity of the question somehow pushing us beyond mathematics, or

aspiration and human attainment and failure. Still, if that vast panorama is to be explored methodically there is a prior need for method.”

¹⁹The implicit reference is to the movement within the second stage of meaning towards the luminous hope of the third. A context here, echoed regularly when I speak of luminous darkness, is P.McShane, “Asia Una Obscuridad Luminosa de la Circumstantia. *Insight* despues Cuarante Annos”, *Universidad Philosophia* (32) 1999. The English version, “Towards a Luminous Darkness of Circumstances. *Insight* after Forty Years” is available on various Lonergan Websites.

²⁰Obviously we are skimming past the power of Cantor's intellectual biography, the story of “one who breached what is, after all, the central fortress of mathematical analysis” (E.T.Bell, *The Development of Mathematics*, McGraw-Hill, 1945, 277). But I would like to think of your psychic orientation - the topic of chapter 10 below - as opening you to slow and quiet delight, apply to this little proof what Bell says of a related proof: “Cantor's proof is in the medieval tradition of submathematical analysis. It would have convinced and delighted Aquinas” (*Ibid.*, 275).

²¹You will find this question freshened in the final section of this chapter.

²²Would Aquinas, or Ignatius of Loyola, delight in this discernment of a core inertia of the institutions of envisagement? So, there are orders of democracy beyond the countability of votes; there are orders of Christianity beyond present stale fixities. The struggle of the present book, as in all my previous effort to promote the order of Hodics, is likely to be a struggle against your inner word of theological order that has possessed minding for centuries. How, then, goes it with you, in this strange nudge beyond seven centuries of decline?

mathematics beyond itself, or mathematics towards a fuller mathematics?²³ But let us be sensitive - a curious demand - to the push, in its line-by-line nudges. Back to the text: “You are beginning to appreciate the flexibility and variability of insight....” The *you* of that quotation is still, now, the intimate minding you, reading along. But it is still so in a new context, a context perhaps quite foreign to you now, that of the third order of consciousness, of the discernment of discernments of discernments, that of a methodology that is mature enough to have ingested its own genesis: thus, wondrously **self-referent** - the haunting bold-faced problem of this chapter. The context is the presence - incipient but invitingly here²⁴ - of a new control of global meaning, conditioned in its growing in you, in global minding.²⁵ So, on with the text: “a long scientific education may be needed to control the spontaneous anticipation of understanding.”²⁶ The stumbling exercises are needed to reveal its givenness. But let us hold to our task, and claim, self-referentially, **This** stumbling exercise is needed. So, we are back at the next statement, “If the fractions between 0 and 1 can be put in one to one correspondence with the natural numbers, then surely the decimals between 0 and 1 can be put in similar correspondence?” *If Then.....* : the

²³But you may arrive, through the mediation of the view of *Insight* 417[442] on the theoretician’s embrace, which is sublated into the third form of generalized empirical method discussed briefly in section 5 below, at the operative suspicion that the move is towards a fuller mathematician.

²⁴Think, echoingly, of Samuel Beckett’s two last little poems, one of which begins, “go where never before / no sooner there than there always”. See *Cantower* 5, “Metaphysics THEN”.

²⁵I permit myself another twisting of your reaching here in bringing you to pause over the secondary meaning of global: the eye as global reach. See note 39 below.

²⁶The statement illustrates the non-moving viewpoint of this book. It is an echo of “the summit of a long ascent” claim of section 1.2 of chapter 17 of *Insight*, the possibility of us contemplating our own nature only after a historical effort “to bring the virtualities of that nature into the light of day.” But what do we mean by those virtualities, that bringing, that daylight? Has it not something to do with the virtualities of such starter-statements as axioms, of marking, penning in and up, the Eschaton, with sufficient sophistication as *x*, the unknown to be known? Clearly, we must weave in here the virtualities of aesthetics mentioned by Lonergan: “What is needed is a qualitative change in me, a shift in the centre of my existing from the concerns manifested in the *bavardage quotidien* towards the participated yet never in this life completely established eternity that is tasted in aesthetic apprehension” (in a review in *Gregorianum*, 1955). Reflection in this mode leads us to such contexts as those suggested in note 191 below.

form of inference presses itself, self-referentially, into non-self-referent print.²⁷ I would say that one has to have been wooed and won by *theoria*²⁸ to pause over such a diagram as the diagram of fractions, to delight in the twists of its achievement, the holding of these dancing little quiddities of mathematical matter and chemical paper in a line-up of natural order.

The *If* becomes thus a unity of minding, something, as we shall muse over in chapter 2, that is brutally cut back in the child-abuse of teaching geometry.

“Then surely the decimals...” The sureness is today’s stumbling lack of couth. But it is also today’s couth regarding and guarding the fractions. Might you not have thrown up your hands at ordering the fractions? Like so many Lonergan scholars throw up their hands instead of their hats at Lonergan’s ordering of the fractions of theology, or indeed of all disciplines? Nor is this a casual distraction or complaint. It keeps up front the whole drive of our seeming little interest. Lonergan, like Cantor, in his drive into “the central fortress” reached something that Voegelin did not sniff in his volumes *In Search of Order*: he reached something “that would have convinced and delighted Aquinas.”²⁹ Sadly, I think back now to Lonergan’s audience for those brilliant lectures on logic of 1957, battered by a culture that demanded *haute vulgarization*. Lonergan might well have said of his lectures what he said of Ladriere’s book: “Ladriere’s aim is to give you everything you need to be able to follow through this whole question of Goedelian limitations. The more mathematics you know, the easier it will be to read this book. And even if you do not know mathematics, you will need to have what I might call a mathematical mentality, an ability to follow complex trains of arguments that are expressed symbolically, if you are to be able to profit from the book.”³⁰ But I look also to the future. Cantor did not know what he was

²⁷It is as well to notice, as we move along, the by-ways and highways by-passed, ways calling to be weaved into our context, but only “by a commodious vicus of recirculation”. (*Finnegans Wake*, 5) We follow the highway of “The Form of Inference”, Lonergan’s obscure early work, in chapter 2.

²⁸See note 141 below.

²⁹See note 20 above.

³⁰*Phenomenology and Logic*, 50.

doing because he did not inherit Augustine's discovery of his inner word³¹, for there was no science of such inheritance. A hodic future grounds, rolls strategically forward, that *nomos*. But what of today's audience, today's couth?

"Surely". We are fantasizing forward to a surety of a central exclusion of oversights that would cut out such "surelies", but with a painful cutting out of centuries of what is brilliantly identified by Lonergan as truncated consciousness.³² Such truncated consciousness can climb forward on Cantor's and symbolism's achievements to take a stand on minding that is crippled at its core. The stand is not redemptive of consciousness. But what of the stand of truncated consciousness that talks of redeemed consciousness without the couth of that redemption through the suffering of thinking that is the authentic recycling of previous thinking?³³

So, there can be an inauthentic surety about the ordering of the decimals: we trust Cantor's diagonal presentation. And there can be an inauthentic surety about the dynamic of minding: we trust Lonergan's presentation. Our return to the diagonal exercise takes a stand against an uncouth trust that meshes both those trusts into a bogus contemporary sufficiency of foundational minding: "believe what Science says."³⁴ What, then, are you and I at when we muse over the "If, then" of the text, and over the achievement of the so-simple breakthrough?

There is, there was, the inner word of the mathematics of the late nineteenth century, shared by the leadership then and now. What was not shared then, or indeed now, was, is, the hold on that inner word that the intussusception of the primary component of the idea at its heart would bring, "the intelligible root or ground or key from which results intelligibility in the

³¹A relevant pointing of Lonergan is quoted later (at note 95) from *Verbum*, but there are also pointers to follow up through the index of **PL**.

³²See "The Subject", *A Second Collection*, 73.

³³I treat of the suffering of thinking in *Joistings* 8, "Recycling Satisfaction". The context is the meshing of a theology of satisfaction into the communal reach for functional collaboration..

³⁴*Insight*, 417[442].

ordinary sense,”³⁵ in this or that piece of mathematical thinking, in this or that homely venture. What are we at when we diagonalize our way out of the decimal ordering problem? We are exercising a hidden exigence to bring order out of disorder, continuous, but in massive discontinuity, with finality’s 13.7 billion year’s search. The exigence is there, in poet and peasant, and it can home in, strangely dark, in 1874’s diagonalizing achievement.³⁶

What happens in and to our inner word as we pen our way to a misfit decimal? What are the mediations of that minding meshed into a neurodynamic of sum-dreaming? There is the minding that creates patterns of marks, an expression of an accounting of counting that feeds back the inadequacy of the accounting. For we are accounting for fundamental dispersedness, and our accounting itself is dispersed. This decimal, “.12345” is not identical with that decimal, “.12345”, for they are apart. This axiom “x or x implies x” is not that axiom “x or x implies x.”³⁷ But is there not a lift out of the here and there and now and then in our struggle for order, with order, towards order? There is: all there, darkly present, in the primary component of the diagonalizing idea.

That dark presence is a fresh uninhibited presence in the early first stage of meaning; the third stage of meaning is to see a refreshing, but a refreshing that turns forward and inward the sophistications of symbols that cry out for self-reference if they are to be ordered.

Cantor’s theorem is a step on the road. It is identified as metamathematics, but is it not within mathematics, self-referential then? The dancing decimals sing their song there, “we cannot be ordered”. It is a fundamental song of the material universe.³⁸ But the song has the finality of a

³⁵*Insight*, 647[670].

³⁶For accuracy’s sake I recall my editorial footnote 29 on page 52 of *Phenomenology and Logic*: “The diagonal argument, generally attributed to Cantor, is found earlier in Paul du Bois-Reymond, ‘Über asymptotische Werte, infinitare Approximationen und Auflösung von Gleichungen,’ *Mathematische Annalen* 8 (1875), 363-414.

³⁷I introduce this topic in **PL**, note 40 of page 62 and note 2 of page 69. We return to it in section 9 below.

³⁸There is the prosaic expression of this at the end of chapter 5 of *Insight*. Recall note 26 above and add the richer reach of note 191 below.

minded song, a song of human consciousness.

Delight in the Cantor trickery. What pulls the eye, that strange reach of mind,³⁹ down the diagonal? Wherefrom the leap towards the notion and the expression of replacement or substitution, coming eventually to a compact symbolization such as

- member Seq: Seq [sub(a_i + 1 for a_i)], shift(diag to horizontal).

2. Going Formal with Goedel

That last bold-faced statement is not, of course, seriously formal, within, say, a system of arithmetic. But it helps towards a sense of what Goedel moves into in his incompleteness theorem. It does nothing more⁴⁰ than specifying a series of moves, moves that might be made by a well-adjusted machine, apart from the bright eye down the diagonal and the bright idea of adding one and other ideas that I leave you to determine, pin down, nail up.⁴¹

³⁹There is the evident problem here relating to extreme realism that I deal with most explicitly in *Wealth of Self and Wealth of Nations*, chapter 5, “The Inside-out of Critical Realism”, and from a different perspective in chapter 5 of *A Brief History of Tongue*. I suspect that you would find both useful and delightful Richard Feynman’s discussion of the eye in chapters 35 and 36 of his first volume of *Lectures in Physics*. I quote a tantalizing paragraph: “As a matter of fact, people who study anatomy and the development of the eye have shown that the retina is, in fact, the brain: in the development of the embryo, a piece of the brain comes out in front, and long fibers grow back, connecting the eyes to the brain. The retina is organized in just the way the brain is organized and, as someone has beautifully put it, ‘the brain has developed a way to look out upon the world’. The eye is a piece of the brain that is touching light, so to speak, on the outside”(36.2, in volume 1 of the three-volume work, Richard P.Feynman, Robert B.Leighton and Matthew Sands, *The Feynman Lectures on Physics* , Addison-Wesley Publishing Company pb, 1963, but many later printings. I recommend this work as a general background: see later comments on it in chapter 14, in section 4: “Recycling Quantum Mechanics”.

⁴⁰Yet this “nothing more” calls us to the reach for a more luminous control of meaning, a reach towards which this chapter points, and indeed towards which the double frontispiece points. One must labour long and hard to bring virtualities into a controlled brightness. The exercise described in note 151 below provides a concrete example of this type of control of meaning. Note that the reach moves us into philosophy of art, philosophy of history.

⁴¹[A bracketed note should help here. As I note in section 5, the first section fits quite nicely into the mood and drive of the new direction of the book. But now we move into a certain un-realism. There is in fact massive trouble, probably beyond your present perspective, in pinning down this stuff, or in reaching some serious appreciation of the well-adjusted machine, a

No trouble in well-adjusting a machine to handle well-formed formulae or programs. But are you well-adjusted to talk luminously about the pull on the eye, the leap, the expressions, the reading of these expressions?⁴² Are you in luminous control of the strange reach of mind which is not a reach but a presence that puts you in two minds, the wonder and the waiter that is not dumb but is rather the fundamental exigence of material finitude, waiting in neurochemical expectation like a photon-thirsty flower?⁴³ Without that control, communally achieved in this millennium, debates about Goedel and minding could go on muddledly, endlessly, even in a good-will staggering truncatedly round the truth.⁴⁴ Do you like the flavour of J.R.Lucas minding of mind, or can you cut through it in the luminous darkness of extreme realism?

“We can see how we might almost have expected Goedel’s theorem to distinguish self-conscious beings from inanimate objects. The essence of the goedelian formula is that it is self-referring. It says that ‘this formula is unprovable-in-this-system’. When carried over to a machine, the formula is specified in terms of the particular machine in question. The machine is being asked a question about its own processes. We are asking it to be self-conscious, and say what things it can and cannot do. Such questions notoriously lead to paradox. At one’s first and simplest attempts to philosophize, one becomes entangled in questions of whether when one

topic for section 6 below. But read on, and place this reading in the context of a single question: what was going on when Lonergan lectured about Goedel in 1957? Of course, one can ask the same of, say, his lectures on education: what was going on in his audience, for instance, when he spoke of history in that final lecture in *Topics in Education*?

⁴²The question simply carries forward the issue raised in the previous two notes. I place that question in a biographic context in Cantower 9, “Position, Poision, Protopossession”. I would note that there is was emphasizing, in the topic of protopossession, the possibility of community. Here I am emphasizing the solitary foundational search that mediates community. Further, I would note that that Cantower was written by a younger me of seventy, only beginning to grasp e.g. the significance of chemical imagery for the control of personal cosmic meaning.

⁴³The mood of chapter 3 of *Lack in the Beingstalk*, can help here, as well as the mood of Cantower 2, “Sunflower Speak to Us of Growing”.

⁴⁴The Introduction to **PL** brings out the massive failure of Logic and Phenomenology to cope with the meaning of truth. It is worth noting that the book *Insight* climbs slowly, into section 2 of chapter 17, towards that meaning. Chapter 2 here will add a few pointers.

knows something one knows that one knows it, and what, when one is thinking of oneself, is being thought about, and what is doing the thinking.”⁴⁵

My position on the possibilities and the probabilities of the needed change is, of course, that it shall come, perhaps in this millennium, not through readings of *Insight* but through the merciless cycles of hodic cleansing. That hodic cleansing should slowly twitch generalized empirical method into its third mode⁴⁶ in a community of the Dark Tower.

The main point is that basic obscurities regarding minding should be met with Cantor.⁴⁷ The obscurities of the Goedelian struggle are a sufficient task without that general cloud of unknowing. But having said that, let me behave contrarily by leading us now on a ramble into Goedel’s Cave. For, the basic obscurities will occupy us through these first three chapters.⁴⁸ And it seems best to begin that rambling reflection by picking up on Lonergan where he ends his

⁴⁵J.R.Lucas, “Minds, Machines and Goedel”, originally published in *Philosophy*, XXXVI, 1961, 112-127. Reprinted in *The Modeling of Mind*, K.M.Sayre and F.J.Crosson, Notre Dame Press, 1963, and in *Minds and Machines*, ed. Alan Ross Anderson, Prentice Hall, 1954.

⁴⁶[This reference, of course, makes no sense without leads. Part of the strategy of the reorientation in section 5 below is to make this mode central to present searching.]

⁴⁷We touch on such basic obscurities in this chapter. Section 9 lifts us to consider the lower grounding obscurity. But of course the entire ramble of the book is around the obscurities that haunt contemporary thinking.

⁴⁸I suspect that you are already sufficiently discomforted by obscurities. Yet there is method in this madness, this play of print and suggestion. But the play of print is needed in its existential obscurity if we, you and eye, are to head towards the optimal goal of this chapter: some fresh sighting of the character of the feeble openness of molecular minding. There you have it, with its self-referential twists: an eye-full which you may feed back linguistically, boldfaced. **Some fresh sighting of the character of the feeble openness of molecular minding.** You, course, are the character, hunting like history for a fuller grasp of the given in its possibilities of patterning. Perhaps it would help to think of these three first chapters in relation to chapter 13 of *Insight*? These chapters move from experiential objectivity to absolute objectivity to normative objectivity: but not firmly. We are on the trail of “the field of the given” (*Insight*, 382[]) but with a terrifying twist towards grasping “the given of the field” where “the field is *the* universe but my horizon defines *my* universe” (PL, 199) [You can notice in this footnote my own discomfort with what I was attempting in the original direction of the book. Again, I ask you here, as I did in the original text above, to reflect on the parallel with Lonergan’s second day of lecturing on Logic (PL, chapter 2)]

comment on the diagonal process associated with Cantor. It is worthwhile, part of sensing of Lonergan's reorientation of Logic and its reception, to place yourself in that first audience who listened to that second day 's lecture. But first, a pause over the opening words of the day found, in fact, at the end of the published version of the first day. They need brooding over: we return to them in chapter 10, when we face the question of biography within the third specialty.⁴⁹

“There is a multiplicity of systems This multiplicity does not come out clearly in most introductory books of symbolic or mathematical logic. But it is a point of extreme significance to us because it brings to light for us things that we would not otherwise suspect, matters that the symbolic logicians themselves never suspected when they set out on their enterprise. It also reveals limitations to their undertaking that provide us with an opening. In other words, it provides a means of leading them on to something that, from the philosophic viewpoint, most of them would not think of acknowledging. The matter is in itself extremely complex and difficulty, and all I am going to attempt to give you is a general outline, an indication, some insight into the sort of thing that happens.”⁵⁰

My footnote comment, below, on that text, locates larger problems lurking here to which we return later, especially in the final chapter. But at least note - and perhaps psychically intussuscept - how the difficulty he mentioned haunts the entire set of lectures, haunts perhaps your, and the original audience's, reception. So we move now to a haunted paragraph.

“All the Goedelian limitations are tied in with this type of demonstration in mathematics, and the tie in is our third step. The logical formalization is given a mathematical interpretation. If the mathematics is sufficiently advanced to include non-enumerable sets, the formalization will include propositions in which you prove that such and such a number within the set, say, k , is not enumerated. LF_2 , as given a mathematical interpretation, will, if it is sufficiently advanced mathematics, contain the proposition ‘ k is not enumerated.’ The argument resembles the previous consideration. We considered the possibility of this list containing all the infinite decimals, and

⁴⁹[That return, of course, will be different from what I envisaged when I wrote the text above.]

⁵⁰PL, 36-37.

we constructed one immediately that was not contained. So we have the proposition, ‘k is not enumerated among the others.’ We have this true proposition, ‘k is not enumerated.’⁵¹

Unless, you are already in the ball park, this is a pretty obscure mouthful: does it mindfill significantly, and if not what are we to do about it and its cousins in culture? I am going forward now to do somewhat the same with Goedel’s work as Lonergan did, but with a twist, a twist which is a poor pre-systematic version of the eventual transformation of language that will constitute mature linguistic feedback Writing about the twist as we move along is a necessary part of our presenting twisting, discomfoting twisting or comfortable twisting.⁵²

The context of my twisting is the twist towards a new context, a twist that is to become an operating vortex of collaboration that would ground a systematico-foundational community in a cherished remoteness of meaning. That community is eventually to be matured and constantly rejuvenated by a quite different world of school education, a world of self-possession governed internally by the third definition of generalized empirical method.⁵³ As Lonergan wrote in 1942 - was it laconic? - “but we are not there yet. And for society to progress towards that or any other goal it must fulfil one condition. It cannot be a titanothore, a beast with a three-ton body and a ten-ounce brain.”⁵⁴ Can you come fantasize with me about grade-twelve girls and boys, in a hundred years or so, giggling over an old book shared by middle-aged folk puzzling over such simple hodic truths as Goedel’s?

So, my patient post-teen reader, let us ramble on. How am I to do this in a way that both

⁵¹PL, 53.

⁵²A curious parallel should help here. I am thinking of something I read in Laurens van der Post, about the elder woman telling the tale of the tribe in the twilight, and the listeners bathing in the mystery. They have heard it before and will hear it again in its mystery. Lonergan is telling the tale of the tribe here, but history has moved us on, ambiguously. There is the deeper need for twilight mystery emphasized in the first section of chapter 17 of *Insight*, a need now for a new street-culture of presentation and reception that meshes us, for example, with the lonely heart-beat of contemporary song.

⁵³See below, in section 5.

⁵⁴*For a New Political Economy*, 20. This is the beginning of a magnificent paragraph that one might well read as a longing or a reach for what I call *The Tower of Able*.

supplements Lonergan's effort and helps us forward in our larger adventure? You recall, I hope, that this is the beginning point of the present book. Was Lonergan clear, in detail, about what was going on in the final twists of Goedel's theorem? If he was he would surely have told me, not asked me, about them, in that conversation I had with him when I was about fifty years old and he seventy eight.

How am I to move on, with you, where *with* has a meaning different from the five days of **with** in those lectures of 1957? Doubling the length of popular presentation just does not do it, unless that doubling turns you to the contemplative shift that this book is about, a shift that I have written about as beyond Zen and Ken to Ven,⁵⁵ strangely echoed for me in the seventh word of what I suspect was Samuel Beckett's last poem. "go end there / where never till then / till as much as to say / no matter where / no matter when"⁵⁶

And why not just put in here a piece of Goedel's theorem, theorem-scarem, which you can read, listen to, in a *Waiting for Goedel* mood? This is not straight Goedel: simply me messing along in a parallel style, pretty close to a popular version of Goedel's Theorem that we shall avail of later.⁵⁷ **(x), - Dem (x, y) is a symbolic metamathematical expression of the meaning, for all x, it is not true that there is a sequence with Goedel number x that establishes the formula with Goedel number y. Briefly, the formula is an expression of the claim that The formulas whose Goedel number is y is not demonstrable.**

The Goedel number of this formula is a number which we call m.

Next consider an activity that gives rise to a formula with Goedel number named sub(v, W, v). That named activity is replacing in the formula whose Goedel number is v, the variable W

⁵⁵My original word here was THEN (see Cantower 4, "Metaphysics THEN", but *Ven* is neater, and gives an orientation within *praxis* towards the dark twilight of the next millennia. And there is the religious twist: "ad**ven**iat regnum tuum". My change of word seems to weaken the reference to Beckett, yet does it not lift Beckett into a new context?

⁵⁶Beckett's last two poems bracket the meaning of Cantower 5, "Metaphysics THEN". See note 24 above.

⁵⁷Ernest Nagel and James R. Newman, *Goedel's Proof*, first published 1958, reprinted 1998, Routledge, London pb. Referred to later as **Goedel's Proof**.

with the numeral for v. and the result of this activity is a new formula with Goedel number $\text{sub}(v,W,v)$. Let us do this activity in relation to the formula

$(x), \text{-Dem}(x,y)$

whose Goedel number is m .

The end result is a new Goedel number: $\text{sub}(m, y, m)$.

What is $\text{sub}(m, y, m)$?

It is the Goedel number of the formula got by replacing the variable y with the numeral m in the formula whose Goedel number is m .

The formula so named and numbered $\text{sub}(m, y, m)$ is

$(x), \text{-Dem}(x, \text{sub}(m, y, m))$.

We now have the following situation: the statement, the formula whose Goedel number is $\text{sub}(m, y, m)$ is not demonstrable, which means *the formula whose Goedel number is $\text{sub}(m, y, m)$ is not demonstrable.*

And with this statement we have the knowledge that *the Goedel number of this formula is $\text{sub}(m, y, m)$.*

What does this add up to, or seem to add up to?

You can grasp, I hope, that it seems to add up to the claim

This formula is not demonstrable, where ‘this’ refers to the formula itself.

It is worthwhile to quote here Goedel’s brief descriptive introduction to his paper. If nothing else it allows you to make the impressive claim in learned company that you read some of Goedel’s famous paper! The quotation is in his terminology but you can slide over that problem to get the points relevant here.

“The analogy of this argument with the Richard antinomy leaps to the eye. It is closely related to the ‘Liar’ too; for the undecidable proposition $[R(q); q]$ states that q belongs to K , that is, by (1), that $[R(q), q]$ is not provable. We therefore have before us a proposition that says about itself that it is not provable [in PM].”⁵⁸

⁵⁸There are two convenient re-publications of Goedel’s original paper (“On Formally Undecidable Propositions of Principia Mathematica and Related Systems”, published 1931): *Goedel’s Theorem in Focus*, edited by S.G.Shanker, Routledge, London and New York, 1988,

Goedel adds immediately here the following footnote. "Contrary to appearances, such a proposition involves no faulty circularity, for initially it [only] asserts that a certain well-defined formula (namely, the one obtained from the q th formula in the lexicographic order by a certain substitution) is unprovable. Only subsequently (and so to speak by chance) does it turn out that this formula is precisely the one by which the proposition itself was expressed".⁵⁹

We have met the notion of substitution - and I even used a touch of anticipatory symbolism there - when we mused over Cantor. And **Dem** is not too confusing a symbol. But what could this business "of Goedel number of a formula" be about? It is both amusing and enlightening to see what Lonergan said on Goedel's strategy in those logic lectures. "The argument is enormously complex, because he has to be able to set up a symbolism for doing the theory of mathematics and a symbolism for doing the theory of logic, and he has to have the two symbolisms running pretty well parallel, or to find a way of communication between the two. That is where the Goedelian numbers come in."⁶⁰ And where Lonergan bows out: there is no gap in my edited text here. He simply rolls on into the topic of transfinite induction, a problem of presentation that Nagel and Newman avoid.⁶¹ But they do give a quite palatable 17-page account of the curious numbering of signs and statements and proofs that does what Lonergan says.⁶² What might I do here to help? This is a much larger question that might appear. Indeed, I would claim that it is a large part of "the problem of general history, which is the real catch."⁶³ The obvious help is to invite you to struggle with the 17 pages of **Goedel's Proof**, or something equivalent. But I can help a little more than Lonergan by giving you a flavour of the operation.

17-47; *From Frege to Goedel. A Source Book in Mathematical Logic 1879 - 1931*, 596-617. I reference these two works as in the next footnote.

⁵⁹**G-Heijenoort** 598; **G-Shanker**,**19**, 42.

⁶⁰**PL**, 57.

⁶¹See **Goedel's Proof**, 97, note 30.

⁶²**Goedel's Proof**, 68-84.

⁶³Lonergan, *Topics in Education*, University of Toronto Press, 1993, 236.

Let's say that you want to get a unique number for the metamathematical statement - it is sort-of axiom of elementary arithmetic as thought out by Giuseppe Peano in 1899 - $(\exists x)(x = sy)$. In plain language the symbolic statement means that from every number x there is a successor y .

Goedel gives every sign of the logic a number so that he can write down a string of numbers for this statement. Here I put the numbers under the signs:

$$\begin{array}{cccccccc} (& \exists & x &) & (& x & = & s & y &) \\ 8 & 4 & 11 & 9 & 8 & 11 & 5 & 7 & 13 & 9 \end{array}$$

Work here towards grasping that this does not give a unique number for the entire formula. Either pack them together or multiply them: each operation gives a single number, but it is not going to be special to that statement. Can you figure out why? Are you going to? Or are you in a hurry? Are you a problem of general history?! And are these questions disturbing, to be dismissed? "We may dismiss them lightly and pass on the the serious consideration of what thought and understanding are in terms of the words that philosophers have been accustomed to use. But we may be wrong in this. We may miss the turning leading to an understanding of understanding."⁶⁴

If you have not skipped, you are a hero.⁶⁵

⁶⁴J.L.Synge, *Science, Sense and Nonsense*, London, 1951, 112.

⁶⁵"If a man is a hero, he is a hero because, in the first reckoning, he did not let the monster devour him, but subdued it not once but many times"(C.G.Jung, "The Relations between the Ego and the Unconscious", *Collected Works*, vol. 7, Princeton University Press, 1966, 173). What a strange sort of heroism it is, to battle with the dispersedness-energy that is constitutive of our comic being. "Why do we honour those who die in battle? A man can show as reckless a courage entering into the abyss of himself" (W.B.Yeats, in a letter). Man, men? For me the reach is symbolized best by that strange woman of music Nadia Boulanger. When asked in 1973 what advice she would give to a young musician, she replied, "Do not take up music unless you would rather die than not do so. It must be an indissoluble love. And one with the great joy of learning, the firm determination to learn, the unswerving perseverance, the intense faithfulness. But primarily if it is not better to die than not to do music - then it is an excuse. And if not then why, why?"(quoted in Alan Kendall, *The Tender Tyrant. Nadia Boulanger. A Life devoted to Music*, Macdonald and James, London, 1976, 10.) We desperately need feminine foundations to replace the mess of men.

Goedel obviously did not skip, indeed the immediate acceptance of his strange theorem certainly owed a great deal to his thoroughness.⁶⁶ Let me just give you his next move:

$$2^8 \times 3^4 \times 5^{11} \times 7^9 \times 11^8 \times 13^{11} \times 17^5 \times 19^7 \times 23^{13} \times 29^9$$

Did you read his move there, without my hint here? The numbers in the previous line-up under the symbols are now powers of other numbers, and the other numbers are prime numbers, and the product of the whole lot gives you the unique number that you needed. O.K? Or is the uniqueness a puzzle still?

At all events you should now be able to go back to the boldfaced print and make more sense of the notion of the Goedel number, called m , of \mathbf{x} , **Dem** (\mathbf{x}, \mathbf{y}). And more sense of the result of the substitution that gives us the formula whose Goedel number is **sub** ($\mathbf{m}, \mathbf{y}, \mathbf{m}$). We shall circle round that formula as we move along here, adding contexts that are directions of reflection, of re-direction. The directions will be unsatisfactory, incomplete. Should you follow the fragmentary re-directions seriously, for the months then require and deserve? To that topic we return briefly in the final section here, section 9, on the Last Theorems of Fermat and Loneragan, but it is really the question of the book, of a missing community fo climbers, and so it is a central topic of the final chapter, chapter 14.. I suspect that your bent at present is to read on, get the gist of where this chapter is going. That brings to mind for me my own tendency during my years of science: a new texts always led me to a read to see where it was going. But the read left me in no doubt that there was a year's work ahead. Gist is an Old French word for abode, but it is normally taken now - apart for its legal usage - to mean *the essence* in some sense of those two words. The gist of this chapter is that the gist of it, and of Goedel or Fermat, or of Loneragan's Standard Model is not easily available. But we may give your puzzling a better focus by repeating the end of the boldfaced section above:

We now have the following situation: the statement, the formula whose Goedel number is sub(m, y, m) is not demonstrable, which means

⁶⁶“Goedel, sensitive to the philosophic climate of opinion and anticipating objections to his work, presented his results with such clarity and rigour as to render them incontestable.”(John W.Dawson, Jr. “The Reception of Goedel’s Incompleteness Theorems”, **G-Shanker**, 74-95: p. 75.)

the formula whose Goedel number is $sub(m, y, m)$ is not demonstrable.

And with this statement we have the knowledge that *the Goedel number of this formula is $sub(m, y, m)$.*

What does this add up to, or seem to add up to?

You can grasp, I hope, that it seems to add up to the claim

This formula is not demonstrable, where ‘this’ refers to the formula itself.

I think now of my own past experience with this situation and this claim, with two summers in the early 1960s and with an autumn, 1965, struggle at the beginning of doctorate studies in Oxford. What does this add up to, or seem to add up to? That eluded me: so I was in no position to meet Lonergan’s late puzzling with more than a generic pointer. You can grasp at least that the situation adds up to trouble, a superficial trouble of seeming self-reference, a deeper trouble about - (about) - human minding. To break beyond that trouble is to enlarge The Standard Model.⁶⁷

3. Formulas, Formulations, Fantasy

First, there is the achievement of mapping. The whole aggregate of metamathematical statements is mapped on a set of numbers.⁶⁸ Have you perhaps thought out what precisely is involved in mapping? It is like a transformation, something the like of which you met before in reading Lonergan: or did you skip that terrible chapter 5 of *Insight*, “a natural bridge over which we may advance from our examination of science to an examination of common sense”⁶⁹? This is a troublesome point or nudge, for we are close to the heart of the present crisis of any philosophy or theology that is comfortable in a commonsense mode, sophisticatedly pseudo-theoretic in its

⁶⁷The Standard Model as it should become operative in this century is sketched in chapter 7; in chapter 14 I will deal further with moves forward in that model that relate to the work of the present chapter.

⁶⁸The question that follows above becomes rhetorical in an on-reading, and I add this note merely to draw brief distracting attention to the massive related cultural problem of a reading that is a non-reading. We return to this problem in chapter 14.

⁶⁹I quote from the first paragraph of *Insight* chapter 5.

anti-contemplative stand. This is perhaps the first of a series of discomfoting nudges associated with the claim that the Dark Tower of functional collaboration involves a climb into a theoretic darkness vital for the redemption of street meanings. Let us cherish a sense of that, good humouredly I hope. The good humour bubbles out of the first four words of the following quotation, on mapping or transforming, from Lonergan.

“It is to be observed that transformation equations, operations of transforming, the whole foregoing account belong to higher-order statements to different universe of discourse. Now the relations between different universes of discourse can be stated only in a further, higher-order universe of discourse; in other words, the relations between different universes of discourse regard, not the things specified in those universes of discourse, but the specifications employed to denote the things.”⁷⁰

You see the humour in the writing and the reading of those four words, “it is to be observed”? There are many such unintended (?) witty remarks in *Insight*. That little “Logical Note” requires quite an amount of observing: if it were observed adequately it would wipe out a mass of muddles about relativity. But that is not our present concern, which is observing the complexity of our mapping of the metamathematics of arithmetic on arithmetic.⁷¹ We add to our task of musing the question of what is meaning by observing. But before getting to that musing, one more nudge.

“What is wanted is an intelligibility grasped in the totality of concrete extensions and durations and, indeed, identical for all spatio-temporal viewpoints.

The answer is easily reached concrete extensions and concrete durations are the field or matter or potency in which emergent probability is the immanent form or intelligibility.”⁷²

Again, we have the humour of our intrepid forty-seven year old typist, with his “the answer is easily reached”. But now we must turn to the first stage of our musings, musings that

⁷⁰*Insight*, 148[171].

⁷¹One trail worth following up here is the trail suggested by HaoWang, that of a hierarchy of languages in which “you are able to handle the logical problems of two levels lower down” (PL, 66). You might try to think this out in relation to Lonergan’s orders of consciousness.

⁷²The conclusion of chapter 5 of *Insight*.

point us both towards a later maturity of the functional specialty dialectic and towards your present stand on the issue “what is wanted”. Certainly that is what Lonergan wanted after precisely twenty five years of a climb of self-discovery. Satisfying that want, a psychic orientation meshed molecularly with his religious dynamic, was integral to his *come-about* that leaps out of his typing fingers two years later. You may not want it, indeed you may be way too old for this sort of business: an old girl or old boy struggling in your late twenties to get a doctorate. But history wants it. It is to be the ethos of global hodic endeavor in the fourth millennium and on, perhaps, to the second million years.

And we must be pragmatic about this. You may, indeed, not be in a position to do this climbing, but I would wish you to acknowledge the possibility, the fantasy, that I present. It seems best to begin that fantasy with an old and favorite analogy of mine: comparing Lonergan’s *Insight* to Joos’ *Theoretical Physics*, a book of the same length that I worked with the year before I began reading *Insight*. It is the same length as *Insight*, a recognized graduate text at the time. In previous writings I have drawn out the manner in which one can also regard, recognized, *Insight* as a graduate text, with compact treatments of topics like Hermeneutics being the same length - 25 pages - as Joos’ dense treatment of orbiting particles, about which undergraduate texts abound. So, Joos can write like Lonergan, “the field equations for a uniform medium follow at once,”⁷³ but they follow thus only for one cosy with extensions of Maxwell’s equations.

It will be generations before the undergraduate texts to underpin the climb of *Insight* appear. In the meantime much of the book gives us something like formulae, which when used properly are a type of doctrine, and indeed I have been suggesting for a decade that the book *Insight* must come to be regarded as a doctrinal book, like a mapping. Formulation - one hopes and one teaches towards - emerges slowly, sweatily. In physics there is an ethos of such sweat of the climb up through undergraduate years. In philosophy and theology there is an ethos - the catch of general history again - of common meaning, a myth of present essential comprehension. So, in my day - is it true still? - second, third and fourth year theology were in the same classroom.

⁷³Georg Joos, *Theoretical Physics*, Blackie and Son, London, 1951, 342.

But these are only brief asides that may help towards fantasy. There is a long hard road ahead to lift method and theology to the level of *theoria*, of contemplation, needed to mediate global progress. Nor is it a private road of theology: it is a main road of a massive change in the ethos of disciplinary focus. Six months ago I took time to brood my way through the 200 odd journals available in my local University of British Columbia. Who is taking charge and where is it all leading or being led? “A bureaucracy cannot create”; there is needed “the cultural overhead” that “must lift its eyes more and ever more to the more general and ever more difficult fields of speculation”. “Nor will it suffice to have some highest common factor of culture, to accept the physical sciences but not bother about their higher integration on the plea that that is too difficult, too obscure, too unsettling, too remote. That was titanothore’s attitude to brain, and titanothore is extinct.”⁷⁴

That hard road towards and in a cosmopolis will be taken under the cyclic pressure of global hodic collaboration, but the individual challenge is quite plain in *Insight* in its very novel view of the reach of any serious thinker. “Theoretic understanding, then, seeks to solve problems, to erect syntheses, to embrace the universe in a single view.”⁷⁵ So, I swing back to that claim at the end of chapter 5 of *Insight*. “What is wanting”. But who is to want it? Surely the creative minority? Surely that includes those who regard the cosmos as part of the seamless garment of a cosmic Christ? And surely that first question of Jesus in John’s Gospel has fresh resonances in our times: “What do you want?”

But my fantasy, our fantasy, is reaching out to envisage the new global leisure and contemplation and mind-reading that is to be the second time of the temporal subject, the third stage of meaning, when the Tower of Able will mediate a continuing freshening of global care, where “suffering soars on summer air.”⁷⁶ We return to these issues regularly as we move through

⁷⁴I am quoting here from Lonergan’s powerful fantasy of 1941-2, *For A New Political Economy*, 20-21. Add to this his fantasy then (see the index under *Leisure*) regarding the future of leisure.

⁷⁵*Insight*, 417[442].

⁷⁶I am quoting here from memory a poem by Patrick Kavanagh the title of which escapes me.

these chapters. But let us turn back to Cantor and Goedel and attempt a little mind-reading.

4. Mind-Reading

We slipped away, at the end of section 1, from Cantor, with a formula that perhaps left you uneasy, and we turned aside from Goedel's various formulae abruptly at the end of the next section in order to add a context of both realism of present reading and fantasy of future minding. Back now to present minding: might we reach together towards some glimpse of the transposition of those formula into the realm of mind-reading?

It seems best to get straight into that and leave heavier reflections to the following sections, when we shall have more data to digest. But perhaps it is a help and encouragement to note that what we do here with the single key formula that we focus on from the Goedel stuff can and will be done with e.g. the single key formula that is the statement "judgments of value differ in content but not in structure from judgments of fact."⁷⁷ Indeed, that doing is the program sketched in chapters 2 and 8 here, and I have already noted that chapter 8 is another option for a beginning of this book as well as a strategic option for beginning, with the second specialty, the practice of functional specialization. What are you reading when you are reading "judgement of value"? Are you not mind-reading? At all events, you are the judge of your own strategy in all this: you may well go back and forth here, and round about, as with the normal crossword puzzle. The difference from such puzzling here is that we are reaching, both you and I, for that comprehensive view, and its symbolic control, that is named in the Frontispiece. We are pushing, as embryonic metaphysicians, for the distant goal of "explicit metaphysics. It would consist in a symbolic indication of the total range of possible experience."⁷⁸ That would be a larger formula, an expression of the logic of being. Where is that logic? Perhaps the full reach of our enterprise can be sensed by observing the fundamental question, "What, then, is being?"⁷⁹ And juxtaposing

⁷⁷*Method in Theology*, 37.

⁷⁸*Insight*, 396[421].

⁷⁹*Insight*, 642[665].

the parallel question of that same chapter “What, then, is critical method?”⁸⁰ The reply that Lonergan gives immediately to this latter question is, “it is method with respect to the ultimate, method applied to the most basic issues”. The basic issues are strangely and freshly present in

(x), - Dem (x, sub(m, y, m))

Come with me, please, in a fresh embrace of the universe. You have seen this print before, above, in the larger boldfaced display. So it is recognized, at least a pattern that might well have been used to condition Pavlov’s dog. The pattern, for us, is a pointer infinitely⁸¹ remote from dog-sight, so evidently such a pointer that we live in a forgetfulness of its being thus.⁸²

But let us swing past that forgetfulness here, to pick up on the boldfaced connection that makes this brief boldface line odd. Its oddness, a major achievement of a great and detailed thinker, is that it has the appearance of talking about itself as non-demonstrable. It has this appearance because sub(m, y, m) is its Goedel number and this is the Goedel number referred to within itself - I leave you to muse over the difference between a numeral and its referent - in its claim that the formula with this Goedel number is not demonstrable. Perhaps another articulation of the problem would help. Let us call the formula, whose Goedel number is sub(m, y, m), *G*. I quote, with slight shifting of terminology to keep our talk uniform, what Nagel and Newman have to say.

“We must remember that the formula *G* is the mirror image *within* the arithmetic calculus

⁸⁰*Ibid.*, 685[708].

⁸¹Various Faiths support this infinity but to get beyond belief is a slow self-tasting climb. “Intellectual development ... reveals ... a universe of being living not as an animal in a habitat.”(*Insight* 473[498]) A great deal - a life-style - of self-attention is required to walk with a dog in the woods or sit with a cat by a fire and sense the infinity.

⁸²The forgetfulness, certainly a topic in existentialist thinking, is very human, especially after history’s break from compact consciousness, and furthered by the axial establishment of truncated consciousness as a way of un-life. While this present chapter relates forward to possible shifts in the Standard Model at the end of this century, it is of immediate significance in challenging that forgetfulness in pragmatic ways: there is the challenge of doing what I call the “Helen Keller” exercise and there is the broader challenge to shed an ‘everyday obviousness’ supported by general bias hiding behind sophisticated language. Such exercises as that pointed to in note 141 offer a discomfiting climb towards and (about)³ the human in us.

of the meta-mathematical statement: ‘The formula with Goedel number $\text{sub}(m, y, m)$ is not demonstrable’. It follows that the *arithmetical formula* ‘ $(x) \text{-Dem}(x, \text{sub}(m, y, m))$ ’ *represents in the calculus the metamathematical statement*: ‘The formulas ‘ $(x) \text{-Dem}(x, \text{sub}(m, y, m))$ is not demonstrable’. In a sense, therefore, this arithmetical formula G can be construed as asserting of itself that it is not demonstrable.’⁸³

This, I hope, nudges you to think of an earlier quotation from Lonergan which had best be repeated here.

“It is to be observed that transformation equations, operations of transforming, the whole foregoing account belong to higher-order statements to different universe of discourse. Now the relations between different universes of discourse can be stated only in a further, higher-order universe of discourse; in other words, the relations between different universes of discourse regard, not the things specified in those universes of discourse, but the specifications employed to denote the things.”⁸⁴

‘We must remember’; ‘It is to be observed’: indeed, yes, there is a great deal of remembering and observing involved here, where remembering is somehow an originating Proustian achievement and observing has the oddity of a Joycean “thought through my eyes.”⁸⁵ In my own case these two paragraphs, parallel in their references to strange mappings, boggled my mind, each in its own context, for many months. How might I help you climb?

At least let us notice where we are or try to be .We would seem to be on the edge of an adventure in a third order of consciousness. But let us be quietly honest with ourselves and with the visible, “the ineluctable modality of the visible: at least that if no more, thought through my eyes. Signatures of all things I am here to read.”⁸⁶

⁸³**Goedel’s Proof**, 69-70.

⁸⁴*Insight*, 148[171].

⁸⁵James Joyce, *Ulysses*, Penguin, 1986, 31.

⁸⁶*Ibid.*

Might I suppose that you are in the Position, the Poosition?⁸⁷ Then you are in luminous possession of the impression that the patterns of print make upon you. That would be a giant advantage in our communication here, a massively “elitist” homeliness that is to be an enabling common presence in the future Tower of Able. But even without that homeliness we can struggle forward here, shackled to some myth of realism. We can embrace G, one way or another, like a Proustian cup of tea, luminous in our longing to taste, to self-taste.

Self-taste? Our seeing of G goes beyond the seeing, Mobius-strip-wise, to a meant-identity, a mind-reading that is at least minimally so.⁸⁸ And minimally self-tasting: on the edge of first order consciousness. We have that state in common with all those who read G, however uncomprehendingly. Let us move on, reaching, however stuntedly and stumblingly, for a frontispiece comprehension of the whole, a whole with its signature of all things I am here to read.⁸⁹ I? Self-tasting, not as immortal diamond but as integral neurochemical receptor of patterns of print, better than a beast in an odd neuroskin named mind.⁹⁰ And our task here is to read that mind through its own slim presence-power.

⁸⁷This is a very existential question, one I dealt with in *Cantower 9*, “Position, Poosition, Protopossession”. In **PL** Lonergan is quite eloquent, especially in talking of Augustine { see the index }, about the problem of reaching the Position. The Poosition is the psychic shift that leaves one’s neurodynamics resonant with the Position. Protopossession relates to the emergence of a community thus psychically orientated.

⁸⁸As a counter image to “inside-outside” problems I have occasionally found it useful to think of, or present, consciousness in terms of the one-sided surface that is the Mobius strip: it is easily formed by twisting a length of paper through 180 degrees and joining the ends, thus arriving at a one-sided surface.

⁸⁹You may think here of Lonergan’s 7½ -page-long climb towards the “Ultimate Significance of Goedelian Limitations” (**PL**, 59-66) with his broad conclusion which we are seeking to refine: “In other words, the human mind, as St Thomas says, has a natural desire for the beatific vision; it is infinitely open. (See note 50 there) That openness is something that upsets this effort, the initial logical ideal of starting out from a whole set of axioms from which you deduce everything that is to be known”(PL, 66). See below, note 129.

⁹⁰A pedagogical context for reflection on this is chapter 40, “Poetics and Constitution,” of *Introducing Critical Thinking*, (John Benton, Alessandra Drage, Philip McShane, Axial Publishers, Cape Breton, 2005).

The task is mightily encumbered with many-tongue myths: are we to think **of** our mind, our minding, this formula? But mind is somehow a skin that thinks **out**, not **of**: but the **out** is sheer metaphor.⁹¹ Have you done the past-modern hunting?⁹² And it thinks out-of from a horizon skinful: “the conceptualization of understanding is, when fully developed, a system and one must advert to the Aristotelian and Thomist *quod quid est* if one would grasp the precise nature of the concept; the concept emerges from understanding, not an isolated atom detached from all context, but precisely as part of a context, leaded with all the relations that belong to it in virtue of a source which is equally the source of other concepts.”⁹³

This is enormously difficult work, as we shall see better from these next five Background chapters, but it is as well to be up-front about the present adventure. Yes, it is a quite small private venture, something you might be daft enough to spend, as I did, two summers of hunting. But it is important to mention my wish to illustrate in this book the manner in which one swings something new into the full hodic cycle, much as the neutrino or the quark were twisted into the cycle of physics in the twentieth century, swelling the standard model of the time. What my “standard model” is, that is a topic of chapter 7.⁹⁴ That new-swing lifts up present possibilities and probabilities of genuine self-attention and self-taste, something shocking and surprisingly rare among those who read Lonergan. The Goedel stuff give us a new nudge, and it is this nudging that calls to us here. So, your present system may be quite remote from the standard

⁹¹A context here is Aquinas, *Contra Gentiles*, book 4, chapter 11.

⁹²Chapter 14 will reflect on the possibilities and probabilities of that hunting. But each of us must discern our own possibility of what I might call “Aristotelian madness”. The notes of the end of this fourth section climb towards note 99's more precise description of that madness.

⁹³Lonergan, *Verbum: Word and Idea in Aquinas*, University of Toronto Press, 1997, 238.

⁹⁴It is perhaps worth musing over what I have been fantasizing here: functional specialization as involving a common sophisticated systematics that is operative in all specialties, reaching upward in the Tower of Able, in a “Calculus of Variation” that has matured much as the zone of physics of that title has matured in the past two centuries. I would recall that Husserl’s work under Weierstrass (1882) dealt with a piece of that history and gives us a good analogue for such maturing. Chapter 4 of *Lack in the Beingstalk*, “The Calculus of Variation”, deals with the topic.

model of chapter 7, but there is a way in which you may find yourself back in a fresh illuminating reading of chapter 1 of *Insight* even now, nudged by this obscure print. We relate freshly to Augustine. “It is time to turn to Augustine ... Naturally enough, as Augustine’s discovery was part and parcel of his own mind’s knowledge of itself, so he begged his readers to look within themselves and there to discover the speech of spirit within spirit, an inner verbum prior to any use of language, yet distinct both from the mind itself and from its present apprehension of objects.”⁹⁵ I am not so ambitious. Sufficient, perhaps, for many of my readers that they discover that they have not been doing what Augustine asked, really attending to that skin of chemodynamics that somehow cherishes and embraces such patterns as **G** in that private kingdom that we name puzzledom.

Augustine was an evolutionary sport, well before the Bell-curve self-attention of “the second time of the temporal subject.”⁹⁶ But **G** points us to a fresh grip on the dispersedness that is the home of spirit, or as I prefer to call it *minding*.⁹⁷ The ramble above, supplemented whatever way you can by other eyefuls, gives us our own mindful of **G**. What is that mindful in you, in me, in Goedel, in the range of searchers since 1931, or perhaps before that? ⁹⁸

The context of the two quotations above, at notes XX and XX,[?V.6,238?] helps us to come to grips with mapping as a minding enterprise:

(**x**), - **Dem** (**x**, **sub**(**m**, **y**, **m**)) corresponds to **sub**(**m**, **y**, **m**),
and in a higher order of discourse we ask what that means. But we ask here What that higher

⁹⁵ Lonergan, *Verbum: Word and Idea in Aquinas*, University of Toronto Press, 1997, 6.

⁹⁶See Lonergan, *De Deo Trino I. Pars Systematica*, Gregorian University Press, 1964, Quaestio XXI. A translation is to appear shortly in the *Collected Works*.

⁹⁷*Insight* ‘s discussion of the spiritual (section 4.3 of chapter 15) needs massive existential elaboration if its meaning is to reach a resonant contemporary vitality. The need relates to a third-stage emergence of a theology of death which is, not a dark theology of termination but an emergentist theology of circumstances of escape, of escape velocity. Thomas struggle with the dynamics of after-life and of eschatology needs creative re-visioning.

⁹⁸A shift in the Standard Model such as is envisaged here grounds a shift in potential discovery of strange searchers, Parmenides types or Zeno types who are tuned exigently and painfully to their own dispersedness.

order is, means, is doing? Are we minding the minding of minding? Yes. But why the layered mess? The layers are measures of the dispersedness that is the floor of the kingdom of puzzledom as well as measures of the upper ground of our loneliness that Lonergan wrote of in **PL**. In the lower ground uncountabilities are like layered new types of decimals, witnesses to the vibrant sameness of continuity.

But let us, piece by piece, seek a minding of the minding of G that we can achieve without the complex context of Goedel. But you can and may continue the struggle to think out and savour

(**x**), - **Dem (x, sub(m, y, m))** , reaching to conceive, with luminous precision, of there being no list of formulae leading to the formula which has that odd Goedel number both within and as a Goedel number. Thinking, savouring? I earnestly join my voice to that of Aristotle, who “went on to urge us to dismiss those that would have us resign ourselves to our mortal lot. He pressed us to strive to the utmost to make ourselves immortal and to live out what was finest in us. For that finest, though slight in bulk, still surpassed by far all else in power and in value ”⁹⁹

5. Back to the Preface, on to an About-face.

Most likely, you and the present generation of philosophers, of *Insight* -readers, will not do this.

So we come to a strange halt to our writing and reading. But only a halt, not a cancellation of the effort so far. I do not cut away those four first sections which were in fact written before the decision written of in the Preface. They are worth climbing through either to the discovery that you are fairly lost in this Goedelian twisting or that you are one of the very unlikely ones, managing quite nicely in this new control of meaning, even perhaps eager now to push on into a

⁹⁹Lonergan, “Mission and Spirit”, *A Third Collection*, 27. I hope that my position in all this is clear enough by now. I am talking about thinking, not something mystical, molecular thinking that keeps pace with the finest global efforts of human arts and sciences, thinking towards that massively disconcerting “come about” that leaves the elder in a strange self-appreciation of self as a blind chemicality weaving tentatively forward in energy’s lonely lovely billion year trek that, while cosmic and communal, is nonetheless a Focused clasping of a unique thread in that spread that is a “song of the Beloved” (see chapter 5 of *Process: Introducing Themselves to Young (Christian) Minders*), as well as Cantower 21, “Epilodge”.

fuller grip on dispersedness' grip on your grip on being.

Your reading of those four sections reveals to you my own struggle with what might be taken as a mix of *haute vulgarization* with bits of encouragement and pedagogy. Aspects of my own re-reading appear in the square-bracketed footnotes. They, and the Preface, suggest, indeed point to, the deliberate taking up of a change of pace: the unlikely turn to thinking about and with the unlikely few. But please don't abandon the chapter or the book immediately: you may well find that it takes a turn to the readable. Recall the point made in the Preface regarding teaching a first course in mathematical physics. It is best done by interspersing the grim struggle of new steps and hard exercises with deviations into larger views, hopes of future vistas. There is to be something similar here.

How might I help you forward with a tolerance of the Existential Gap?¹⁰⁰ It seems to me that a broad help is found in presenting to you briefly something prepared for a particular conference, that of August 2006 in Vancouver on "Research and Communications": a brief doctrinal identification of the three modes of generalized empirical method. And I would have you focus your attention, your self-attention, on the third mode that I come to at the end of this piece. It is a mode that is to be at the heart of living in the third stage of meaning, whether the living of common sense, or aesthetics, or theory, or prayer. I would like to think that the stretching that is the remainder of this book brings forth a response like that of Debussy to Stravinsky's *Rites of Spring*. For months he struggled with its new twists but finally wrote to Stravinsky: "For me it is a special satisfaction to tell you how much you have enlarged the boundaries of the permissible in the empire of sound."¹⁰¹ And I would like to think that the new twists might be identified with the exigence constitutive of your loneliness for a form that is both a grasp of itself and the source of the concrete events that is this pattern of print, those numbers of Goedel, that heart of darkness that is our world of rich abstraction. So you could find yourself eventually reading with the freshness of a dream Lonergan's radical completeness and incompleteness theorem of chapter 19 of *Insight*:

¹⁰⁰See the index of **PL** on the topic.

¹⁰¹Quoted in Donald Mitchell, *The Language of Modern Music*, London, 1966, 22.

“An unrestricted act of understanding will understanding everything about everything with no further questions to be asked ... To understand each concrete pattern entails knowledge of the totality of events relevant for each pattern, for the concrete pattern includes all the determinations and circumstances of each event. Nor does this contradict our prior conclusion. For the unrestricted act of understanding proceeds, not from a grasp of abstract systems of laws, but from a grasp of itself.”¹⁰²

Goedel’s Theorems lead there, and the two volumes of Penrose, but only through the gateway of the third mode of generalized empirical method. Let us then at least get a nominal grip on that feature of The Standard Model, a feature that is to embrace the globe in later times. The gateway, oddly, is not deeply remote, but in our times it is closed to the vast majority by the cultural dominance of truncated subjectivity, even with Lonergan studies.¹⁰³ But let us venture into the elementary pointers of the Vancouver Conference:

Generalized Empirical Method, Third Stage

1. Three Definitions of Generalized Empirical Method

First, a creative pointer here regarding the title, stages of method. Think of three views of generalized empirical method as associated with the three stages. Basic spontaneous method is present from the beginning, the early methodologist being the human who has as yet not planted nor harvested not even found a shell convenient for gathering berries. In the first stage of meaning, in its generic purity, attention is on the object: there emerges empirical method, a spontaneity that can invent instruments of survival. It is unanalyzed, but eventually it takes descriptive shape in a talk, a linguistic trick, that leaves out the source of that shaping. There is, then, talk of empirical method that has the characteristics of the later talk, a contemporary talk,

¹⁰²*Insight*, 650[673].

¹⁰³Mark Morelli presents a remedial strategy in “Empirical Consciousness in *Insight*: is our Conception too Narrow?”, *The Importance of Insight: Essays in honour of Michael Vertin*, University of Toronto Press, 2006, 36-87

indeed, that has its screening roots in truncated subjectivity.¹⁰⁴ From that sort of talk and thinking one can arrive at the expression of Lonergan in the third chapter of *Insight*: “We have followed the common view that empirical science is concerned with sensibly verifiable laws and expectations. If it is true that essentially **the same method could be applied to the data of consciousness**, then respect for ordinary usage would require that a method, which only in its essentials is the same, be **named generalized empirical method**.”¹⁰⁵ This may be taken as a first definition of generalized empirical method.

Next comes Lonergan’s later definition of generalized empirical method, that should dominate these next centuries. It still does not seem to have much influence on Lonergan students.

“Generalized empirical method operates on a combination of both the data of sense and the data of consciousness: it does not treat of objects without taking into account the corresponding operations of the subject; it does not treat of the subject’s operations without taking into account the corresponding objects.”¹⁰⁶

The Third Definition of GEM is my suggestion, though you can find it lurking in some of Lonergan’s writings: e.g. in “Mission and Spirit,”¹⁰⁷ Or in his view of leisure as an emergent of a new economics.¹⁰⁸

Generalized empirical method still operates within the second definition, but the focus is now on the roots of the operations of the subject, the loneliness that is the heart of history.

Should I leave at that, with the invitation to brood over the two shifts?

¹⁰⁴Recall Lonergan’s comments on page 73 of *A Second Collection*. In his essay “The Subject”.

¹⁰⁵*Insight*, 72[96].

¹⁰⁶Lonergan, *A Third Collection*, 141, top five lines.

¹⁰⁷*A Third Collection*.

¹⁰⁸See the index, *For A New Political Economy*, under *leisure*.

Let me see can I give some uncomplicated hints. But I would note that digging out the meaning of the two definitions is a matter of new research into history. So, one finds the third definition verified in a vague way in aesthetic reachings, in primitive poetic yearnings. On the other hand, one finds in the recent history of Lonergan studies a massive neglect - or dodging - of the second definition. Too many Lonergan pseudo-disciples incline to write of conscious operations, say, in physics or psychology, without venturing into the data of sense. Let me be extremely simple here: what data of sense do I wish to draw to your attention? Yes, of course, it is the data that physicists study, the data that psychologist study. But think now of the data that these people produce: print about physics and about psychology. What is being neglected is the mediation of an understanding of the operations that is being made available in history by the venture called the scientific revolution.

The third definition of GEM seeks to carry forward all that mediation of humanity's reach for explanation into a new culture of leisure and luminous loneliness. To fantasize forward about it is a massive foundational undertaking. Suffice it to say that it will lift the meaning of the first section of chapter 17 of *Insight* into a quite new context. *Haute vulgarization* is to be replaced, with statistical success, with a common sense of mystery, human living will reach new levels of privacy that is intimately global, and the mystery of human death will be a mystery of hope.

2. Conferring about Research and Communications¹⁰⁹

It will take us a little work to glimpse better the meaning of the third - or even the second - definition of GEM. But that glimpse will come with hum-drum practical considerations on how the distant aspiration that are in those two definitions can help us towards a discontinuous shift in Lonergan studies and in our own work. As I have been envisaging it, and was going to envisage it here in the original essay, it is quite a fantastic yet obvious shift. It is a lift associated with the weak treatment in *Method in Theology* of the two specialties. Yet it is also related to the

¹⁰⁹The orientation is towards the initiation of these two specialties, but obviously is not restricted to them. Each of us has a bent towards one or two specialties, already perhaps identified, or waiting in our loneliness to be identified. And, of course, the conferring is a foundational search meshing into the tasks of the fourth and fifth specialties.

minimalism that I have been advocating for some time now, and to concrete possibilities and probabilities in what I might call our ordinary lives of marginal scholarship.

This latter minimalism and ordinariness is what the conference conferring is about. Indeed, such is my present minimalism that I do not wish to burden you with readings on previous efforts to say what specialized work in the first and last specialties is. I list some such readings in the last footnote and here and there as we ramble along together, but I do not ask you to follow up on them: I wish you only to follow up on simpler possibilities that come to you either from among those touched on by me here, or that dawn on you through the present nudging of your life.

Still, I presume that you have some notion of my minimalism. Quite simply, it advocates the division of labour advocated by Lonergan without its grounding: grounded rather in noticing the muddled presence of that division in contemporary studies in all serious domains.¹¹⁰ Now, not only do I presume that you have some notion but I also wish to presume that you are taking sides about it, taking a stand on it. What stand do I desire? Here, oddly, I am stepping away from minimalism to the fantastic. At least, viewing current Lonergan studies, it could strike you as something in the realms of fantasy. The fantastic minimalism stand is that what Lonergan suggests is something that could take over the globe, become the dominant ethos of all learning, its sharing, its implementation. This, after forty years of brooding, is not fantastic to me: indeed it was pretty evident to me in the late 1960s. But what is growing ever more evident to me as we move along in Lonergan studies these decades later is that Lonergan achievement has at best a place in scholars' minds as a convenient filing system for the individual. Nor do I see this placement as something they consider to be a temporary strategy.

So, I am asking for a stand on this fantastic minimalism. I can, of course, have a shot at persuasion, and this in three basic ways that can be intertwined. There is the heavy way of dialectic which I have dealt with at considerable length elsewhere¹¹¹; there is the commonsense

¹¹⁰You would find helpful the reflections of chapter 3 of *Pastkeynes Pastmodern Economics: A Fresh Pragmatism* and of chapter 1 of *Method in Theology and Botany*.

¹¹¹The primary group of writings on the matter are the series of Website essays titled SOFDAWARE and Quodlibets, some 200 pages dealing with that single page 250 of *Method in*

way that lurks in my appeal to history or my appeal by illustration from difficulties in various disciplines¹¹², and there is the third way that consists in drawing attention to the manner in which the fantasy fulfils the conditions for cosmopolis set out by Lonergan.¹¹³

Now if you are with me in this stand, even in a commonsense fashion, then we can proceed to envisage strategies that relate to commonsense versions of the specialties research and communications. This should, at first glance, seem odd to you: the specialties in their maturity require subtle differentiations of consciousness. How are we to manage the envisagement while operating in a commonsense mode?

We do so because we hang in with one of the facets of these specialties. We do research, but have no intention of going further: we are like lab attendants in physics, screen watchers on a warship, capable of handing on the baton by saying “hey: look at this!” Similarly, we do Communications but we are not leaning on the massively-developed cyclic support of the future: we are simply saying “hey, look at this!” But note the difference in the Hey-saying. The researcher is nudging those in the community of Lonergan students: the communicator is nudging the general community in particular zones.

But what commonsense helps you to notice what you say “hey” about? It is a business of layers, the identification of which is a task of our collaboration, but in my effort to get us into this task I would have us get thinking about the main characteristic of the commonsense bent that I have in mind, that I wish you to have in mind, in character.¹¹⁴ It is the bent that wishes not only to

Theology.

¹¹²The best reference here is chapter 3 of my *Pastkeynes Pastmodern Economics. A Fresh Pragmatism.*

¹¹³These are well worth brooding over in this context: giving section 8.6 of *Insight* a fresh reading..

¹¹⁴I think of character as defined in the beginning of the Aristotelian *Magna Moralia*(see note 13 above), or the meaning of *character* as mentioned in section 1 of chapter 14 of *Method in Theology.*

see results, but to be the agent of some results.¹¹⁵

This may not seem much to ask, but in fact it asks much when viewed in its fullest sense.¹¹⁶ But lets not go there: think at present of a bubbling up of a commonsense ethos, say, in the midst of a conference on Lonergan, pushing the existential question, the molecules of the participants, towards the question of efficiency ... **Where is this going?**¹¹⁷ For instance, ‘Is this paper that I am listening to going to hit the streets?’. You find this, perhaps, an unfamiliar attitude? An unwelcome, disconcerting attitude? Even more so when the asking is ‘Where am I going with this? Is this leading me, us, anywhere as “a practical view of history?”’¹¹⁸

So we get closer to the mood of our involvement with withdrawal, a withdrawal that I would identify as contemplative, not a prayer of quiet, but an Augustinian “restless heart”, a Theresian adventure.¹¹⁹ And now, re-view the definitions of generalized empirical method in this light and notice new light, a new control of meaning.

But I wish to hold to brevity here. Where are we going with, in, from, this August gathering? Are we tuning to cherishing freshly, pragmatically, cunningly, the loneliness that is the heart of history? Are we ready, “ever ready,”¹²⁰ to make Hey while the Son shines?

¹¹⁵The final section of chapter 3 of *Method in Theology*, with its contextualization of effeteness, is relevant here and I would draw attention to the two comments (pp. 121, 155) on *haute vulgarization* in Lonergan’s *Collected Works*, vol. 6.

¹¹⁶The viewing is the distant reality pointed at e.g. in chapter 4 of *Lack in the Beingstalk*.

¹¹⁷The context of this question should eventually be the unity, beauty and efficiency of the new metaphysics that is serious about efficiency (see *Topics in Education*, 160, line 16) and about implementation as the core of the definition of metaphysics. God’s concept is an eternal practicality: see note 120 below.

¹¹⁸*Insight*, 233[258].

¹¹⁹In *Joistings 4*, on “Personality Types”, I reflect on the three Theresa’s of India, of Liseaux, of Avila. There are deep issues here of the character of contemplative reaching but the generic point is made in *Cantower XXI*, “Epilodge”

¹²⁰*Insight*, 726[747]. A matter, you might sense, of becoming a “specialized auxiliary” with “an effective determination to discover and to implement in all things the intelligibility of universal order that is God’s concept and choice.” God’s concept is the Son, shining in the

The Hey depends on where we are and stand in a common sense, with perhaps a tincture of theory, of our own participation of history's loneliness? So, we must attend together to our opportunities to lift the ordinary of our quest into the rhythms of an extraordinary recycling of meaning that is yet to be, by taking note - Research - and giving notice - Communications - of simple agonies of our classrooms, streets, conferences, collaborations.

Of what do we take note, and where do we take it? Of what do we give notice and to whom? What is your fancy? Certainly I have my own fancy, indeed a massive list of fancies that, in a broad sweep, were expressed in the remote doctrines of my last Cantower.¹²¹ But it seems better to await out interchanges before, during and after the August gathering.¹²²

What I have written of in that piece is an elementary aspect of the about face involved in this millennium's struggle. Might I point you towards more complex aspects of it?

6. Algorithms

We move from a wide-ranging reflection on Lonergan's styles of presentation to Penrose as he presents to us the workings of Turing machines. Penrose introduces the topic by reflecting

darkness of today's opportunity.

¹²¹*Cantower XXXXI* dealt with the functional specialty Doctrines, but it also was the beginning of a new pragmatism that I saw as necessary: so, I ended the million word project after 400,000 words. The doctrines noted there are remote in meaning, but the present move is towards an intussusception of them within common sense. But I would wish that move to be a communal effort.

¹²²The communal effort of our gathering leads me to cut out of this essay my own lengthy listing of patterns of intervention in present fashions of conversations and classrooms and conventions. Still, you might find it useful to check out the pointers towards new twists on Research and Communications in *ChrISt in History*, which is on the Website, or in *Method in Theology and Botany*. But I suspect that we will find surprising the range and number of pragmatic interventions that are possible and probable in a genuine lift of Lonergan's meaning into contemporary highways, lowways, buyways, into the corridors of power and pedagogy.

on the strategy of algorithms.¹²³ Let us replace his illustrations here with a simpler one, appropriately the one that was the topic of Lonergan’s last presentation: the old algorithm for getting square roots.¹²⁴ But I recall our challenge of re-direction. In the concluding section we shall indulge in fruitful fantasy about facets of it. Here the facet is future foundational pedagogy. One might think of it as perhaps filling gaps in chapter one of *Insight*. But let us hold on that topic till we move, for a first time, through the tea-tasting adventure of taking square roots. I invite you to avail of my introductory presentation of over thirty years ago. Of course, you can break off at any stage to solve the problem on your own; or you may just cherish freshly a tea yet to be fully tasted.

“Let us recall, then, the rule for extracting square roots which no doubt you learned in school. A simple illustration will serve:

$$\begin{array}{r}
 1 \quad \underline{1 \ 3 \ 2} \\
 17424 \\
 23 \quad \underline{1} \\
 \quad \quad 74 \\
 \quad \quad \underline{69} \\
 \quad \quad \quad 524 \\
 \quad \quad \quad \underline{524}
 \end{array}$$

You recall the rule? The number whose square root is sought is divided into pairs of numbers starting from the unit end. These pairs are ‘brought down’ successively. Each time the number already reckoned e.g. 13, is doubled and used to determine the next number in the square root etc., etc. A little practice will refresh your memory and the process of getting square roots can become almost automatic. You can use the rule with ease. But our crucial question is, do you understand the rule, the *Why* of it? What is *meant* by the procedure?

{A few paragraphs later in the original text:} I have provided you with a puzzle: what is the meaning of this rule for extracting square roots, why does it work? Now I would like you to solve this puzzle but during the procedure to have a dual interest. For I wish you not only to be

¹²³Penrose, 30-35.

¹²⁴Lonergan’s final appearance at a Boston Workshop, if I recall rightly, included a tired presentation of the “square-root technique” and the need to come to grips with its grounds.

interested in solving the problem, but also to be interested in how you go about doing so. I want you to add to the puzzle the question, ‘How do I go about solving this puzzle?’ You should have, in other words, a methodological interest in the procedure. You should be able to ask and answer such questions as: Am I conscious, wondering, using diagrams, adjusting diagrams (disposing the phantasm, Aquinas would say), getting clues, hints, little insights?

{Another paragraph later:} Now that you have at least tried, if not succeeded, let us consider how understanding of the rule may emerge. Why double the results each time? Why pair off the numbers? Let us perhaps reverse the process, square the result. How? By straight multiplication. Now, let us dispose the phantasm, let us spread it out: the answer, 132, can be broken down into $100 + 30 + 2$, so instead of writing $(132)^2$ we can write $(100 + 30 + 2)^2$. You may not have remembered how to expand that, but with a little juggling you can arrive at the result:

$$(100 + 30 + 2) = 100^2 + 30^2 + 2 \cdot 10 \cdot 30 + 2 \cdot 100 \cdot 2 + 2 \cdot 30 \cdot 2$$

A clue that you might have got at this stage would relate the multiple ‘2’ of the expansion to the doubling of the results at each stage. Notice here how the illustration was chosen helpfully to the extent that 132 does not include a repetition of a number: this helps for identification in the expansion. But notice also that it would have been more helpful to have had an illustration without the occurrence of the number ‘2’, for that ‘2’ can be confused with the multiplying ‘2’. All this is related to Aquinas’s and Aristotle’s notion of modifying the diagram, disposing the phantasm, and to the role of the pedagogue in ‘setting things up’, so that the solution begins to ‘stare the pupil in the face’.

Next we recast the expansion to help you (in case you have failed):

$$\begin{aligned} (100 + 30 + 2)^2 &= 100^2 + 2 \cdot 100 \cdot 30 + 30^2 + 2 \cdot 100 \cdot 2 + 2 \cdot 30 \cdot 2 + 2^2 \\ &= 100^2 + (2 \cdot 100 + 30) \cdot 30 + (2 \cdot 100 + 2 \cdot 30 + 2) \cdot 2 \\ &= 100^2 + 230 \cdot 30 + 262 \cdot 2 \text{ “}^{125} \end{aligned}$$

¹²⁵*Wealth of Self and Wealth of Nations*, 19-21.

My text adds other reflections that you can avail of on the website,¹²⁶ but I would wish to draw attention to a single point related to our re-direction. You notice the mention half way through of a dual interest: it is the dual interest of generalized empirical method in its second mode. But now I wish you to cultivate the third-mode stance, mediated by the second mode interest. It is not an easy cultivation, yet it is to be the mood and the mode of the third stage of meaning, the second time of the temporal subject, and indeed the mood of mature kataphatic contemplation that bends towards God being not-an-object.¹²⁷ It is the mediation of cosmic-caused appreciation of desire.¹²⁸

Nor is that last sentence a distraction: it meshes with the ultimate significance of Goedel's theorem in our twining both with the upper and lower grounds of loneliness.¹²⁹

Have we finished with the square root illustration? One does not finish, no more than Proust finishes with the tea and cake, no more than von Karajan finishes with Beethoven's early symphonies.¹³⁰ But there are deeper unfinished businesses that we had best attend to before taking

¹²⁶The entire book, *Wealth of Self and Wealth of Nations*, is available there; www.philipmcshane.ca

¹²⁷“On what I have called the primary and fundamental meaning of the name, *God*, God is not an object” (*Method in Theology*, 342). I would emphasize, as I do regularly in such matters, that such a proposition is not an exclusive expression of mystical experience. It is a matter of coming to grips thinkingly with the grounds of nostalgia.

¹²⁸Helpful here are the conclusion of chapter 2 of *Lack in the Beingstalk* and the move forward of chapter 3.

¹²⁹See note 96 above, where I recall Lonergan's pointing to the upper ground. The intent in this chapter is to lead us towards facets of the lower ground of loneliness, facets of dispersedness and energy. These would seem to be within the reach of his late question to me. The direction relates to my cheeky conclusion of the end of section 8 below, and to the drive of note 203. Mathematical logic's incompleteness theorems and related problems invited larger precision regarding dispersedness, virtuality, energy, expression, identity, self-identity. Such larger integrative precision is to be, in a hundred years or so, a lift of the Standard Model analogous to the lift sought in present particle physics. It is to emerge out of hodic collaboration.

¹³⁰In the Introduction to von Karajan's second last recording of the symphonies there is quoted a remark of his about the fourth and fifth symphonies. He had spent the summer listening to them more than 200 times in editing his *Deutsche Gramophon* recording and to a friend's

to the hard road, with Penrose, of touring Turing machines. For there are three different insights lurking in our exercise that need attention. There is the grounding insight of language; there is the odd insight that carries us beyond troubling about this '100' being different from that '100', there is the insight - or set of insights - that somehow holds gloriously in humanity's dance of truth. Here is a case, or rather three cases, of a direction that belongs to the re-direction of which I wrote in the Preface. The dance of truth is to occupy us in the second chapter. What of the other two issues? They are cases of a firm direction to go elsewhere, or of a decision to repeat previous texts here. In the case of the grounding insight of language I direct you elsewhere: you can take it as a measure of your seriousness whether you think it worth, say, month of your life. It is an essential of foundational growth, quite obvious as a common component of viewpoint in 22nd century hodies. That 22nd century hodies, furthermore, is to be a cycling and sloping of specialties in all areas that will swing linguistics, psychology, and related disciplines out of the present stupidities regarding the emergence and grounding of human language.¹³¹

I direct you in this matter to two texts: *Method in Theology*, 70, where Lonergan has a paragraph on Helen Keller's leap to language, and *A Brief History of Tongue*, 30-36, where I spell out all too briefly exercises that would bring one to "thoroughly understand what it is to understand" human sign-use.¹³² Recall our reflections on Lonergan's presentations, and note the

suggestion to him that he might have been bored in conducting them three weeks later he replied, "From the moment I began they were new works to me".

¹³¹The problem is the basic topic of *A Brief History of Tongue. From Big Bang to Coloured Wholes*, but it is as well to move through standard texts to appreciate the horror. A standard text that I recommend generally for a venture into neurodynamics is Mark Bear, Barry Connors, and Michael Paradiso, *Neuroscience. Exploring the Brain*, Lippencott, Williams and Wilkins, 2001. This is the central massive oversight in the book (see chapter 20) but one can track that existential gap through any author, modern or post-modern.

¹³²The understanding of the leap to language is certainly central to this thorough understanding and its luminous presence transforms one's presence in the reading of *Insight* or any work, like the defective one's mentioned in the previous note. I would like to take the opportunity here to nudge you regarding the word "thorough" used by Lonergan. Its meaning is tricky, since it includes the luminous darkness central to all human inquiry. That darkness, however, can be focused by inverse insight whether in natural knowledge or in the knowledge of theology. Chapter 5 of my *ChriSt in History*, a Website book, deals with the control of the

doctrinal nature of his paragraph, a paragraph I surmise, that has been rarely read as doctrinal pointing to a difficult climb. Yet without that climb e.g. the entire book *Insight* is clouded. So I would claim that the exercise outlined in *A Brief History of Tongue* is a must of progress in self-tasting, and will be acknowledged as such in a hundred years or so.

What about that other odd insight regarding a 100 here, a 100 there?¹³³ It is the ABC exercise of *Insight*, occurring twice in that book but with quite different meanings.¹³⁴ Another instance of Lonergan's short doctrinal presentations, but certainly not *haute vulgarization*. I try to capture the mood of an adequate exercising regarding this home of human minding in *A Brief History of Tongue*, 148-55.¹³⁵ This is tough work, but it is to become easier in so far as The Standard Model generates a community that shares both Position and Poosition. That section in *A Brief History of Tongue*, and its including chapter, help towards that, and we nudge further below.

That final chapter of *A Brief History of Tongue* is meant as a re-reading of chapter one of *Insight* within, or at least towards, a luminous personal positioning with regard to truth, being, objectivity. A very discomforting business - our third case of lurking trouble - which we leave, as I mentioned, to the next chapter. But now, do we have a problem? Can I presuppose some modestly-successful venture into that three-layered work as a preliminary to the plunge into Penrose's treatment of Turing machines? Well, be of good cheer: Penrose hasn't come to grips

darkness in the field of revealed mysteries.

¹³³In a note to page 62 of **PL** relating to Lonergan's various presentations of the problem I remark: "Lonergan repeats this brief task description, with slight modifications, in *Insight* 51, 527-28, and in *Understanding and Being*, chapter 2, conclusion. Its brevity belies its importance for foundational work. One can glimpse this by taking the axiom 'x or x implies x' mentioned below in note 2 of p. 69, placing the three x's in an equilateral triangle, and pursuing a more fundamental axiom that meshes with the psychology of this-saying."

¹³⁴*Insight*, 26-7[51], 504[527-8].

¹³⁵On page 152 there I gave the original verbatim version of the **PL** text, useful for some extra hints..

with these issues either.¹³⁶ All the more reason for us to congratulate him on his magnificent control of meaning in what he writes in the chapter that concerns us now.

But why should it concern us? The immediate reason is that the venture gives us another angle on the Hilbert project and on the achievement of Goedel. Moreover, this approach helps us connect the Goedelian project with the Cantor strategy already discussed. For you, however, it is a decision-time that parallels the decision time of the first page of *Insight*: taking the direction of serious reading. I associate the next section with the nudge towards attending to small problems given in the first paragraph of the first chapter of *Insight*, and I would certainly like you to pause over that association in what I might call a self-search for a 250-stand. We are reaching for a new perspective on the first chapter of *Insight* and the exercises required to self-shift into an explanatory world. I suspect that not a few readers of this chapter will react badly to my suggestion about getting to grips with chapter 2 of *The Emperor's New Mind*, or an equivalent text. Such a slipping past was regular, during the past fifty years, in the reading of *Insight's* first chapter. There is an optimism in Lonergan's very solitary writing of the book. What could have been on his mind when he concluded the chapter with a broadening of his invitation? "“Instead of the transition from elementary arithmetic to elementary algebra one may review the process from Euclidean to Riemannian geometry. Instead of asking why surds are surds, one can ask why transcendental numbers are transcendental”". I suspect that there was hope in him of reaching a wider audience than the conventional audience of philosophy and scholasticism. But this was not to occur. It is to occur, indeed, through the probabilities offered by the move towards a globalization of functional specialization. I do not wish to repeat the dynamics of that globalization here and the manner in which diverse disciplines converge, slope towards each other, as each moves from research through history to a dialectic searching for their own foundations, discomfortingly found to be common to all disciplines.¹³⁷ Suffice it to say that in

¹³⁶I mentioned this failure in the Preface and will return to it in chapter 14.

¹³⁷A lengthy discussion would be needed to bring out the manner in which these aspects of the operation of cosmopolis achieve the efficiency (see Topics in Education, 160, line 16) of genuine metaphysics against the human potential for shrinkage of objectives. There is an essay to be written dancing round the suggestion that “doctrines that are embarrassing will not be

that future context Penrose is to meet and greet his philosophic or scholastic counterpart and then the scholastic being "a little breathless and a little late"¹³⁸ is to become a quite visible scandal.

But this opens up the large topic of what we are doing now in Lonergan studies, what might we do now, what might be the dynamics of the Standard Model in a hundred years or so. I have not yet, however, detailed that Standard Model nor indicated its functioning in the cycling of specialties. We tackle that in Part Two here, chapters 2 to 13, in the manner indicated in the Preface. That background, even without its serious digestion, should enable us to get some perspective on the academic ethics of the next millennium. We will come to that topic in the fuller reflection of chapter 14 but it might be a help to venture into that chapter even now. The difficulty is motivation, associated regularly with shortage of time. How could I possibly find a month to work on this single topic? And it does take a month or a year: "experto crede"¹³⁹

The Turing problem is, of course, only one possible illustration, self-illustration, of just how seeking understanding works in our feeble humanity, though it is a key bridge, as Penrose knew, as we may come to appreciate. However, you may find some serious problem of your own: Riemannian geometry, as Lonergan suggests?

Why, you may ask, do I stress problems in mathematics or physics? Because these are the simplest zones of inquiry that are at present beginning to shape up in seriousness. As we move up through chemistry to botany and on to theology we enter mythic worlds, illusions of explanatory control. But again, larger topics are bubbling up.

At all events, if Turing isn't immediately attractive, perhaps I might cajole¹⁴⁰ you into

mentioned in polite company" (*Method in Theology*, 299).

¹³⁸*Insight*, 733[755].

¹³⁹I refer here to Lonergan's use of the phrase when he is writing of his long struggle towards his economic theory (*For a New Political Economy*, 112). I think of my own 30-year struggle with that economics, or the forty years that it took me to get a grip on his meaning of **energy**. It takes a peculiar psychology, a committed foundational psychology, to push on towards this intellectual conversion. See note 141 below on one suggested climbing experience which, I suspect, you would find easier than the climbing of this chapter.

¹⁴⁰Need I recall *Insight* 398[423] on cajoling or forcing attention? This little book seeks to cajole attention. History, in its genesis of cyclic collaboration, is to add force. And there is the

tackling the little problem I posed elsewhere, of seating people round a table?¹⁴¹ It is almost a party game, very swiftly, uncomfortably, revealing “the necessity of effective criteria for determining when adequate insight actually has occurred.”¹⁴² But I would wish you to at least take a shot at this next section, with its necessary direction to the back-up text of Penrose.

7. Turing Machines

The direction to **Penrose** is, of course, not really necessary. I could produce my own version here. I suspect that it would be very much longer than Penrose’s thirty pages in that I would supplement his brilliant presentation with lower-level pedagogical nudges. Besides this, we have the task of lifting his efforts and ours into the quite strange context of self-taste, into the third mode of generalized empirical method. Who, what, are we that have such a hard dispersed time in reaching insights that are beyond naming? We come face-to-face, behind our eyeballs, with thirty pages that point us to a minding of very elementary machines and their output. What is that minding in you and me, and what is its trail, its cosmic trail, and what does it reveal to us of our loneliness, our Proustian reach for taste, our Hopkins-type reach for self-taste, our restless Augustinian hearts?

So we move from our struggle with the algorithm for square roots to Penrose’s first demand. “Let us try to imagine a device for carrying out some (finitely definable) calculation

discomfort of objectified embarrassment: see note 147 above.

¹⁴¹This could be, for you, the most important footnote in this chapter. It brings you back to the challenge of the first paragraph of that first chapter of *Insight*. It could save you from the failure of your ‘longer cycle’ education which regularly blocks people from any sense of serious understanding. The exercise involves only elementary mathematics. Here you have it: “How many ways can n married couples be seated about a round table in such a manner that there is always one man between two women and none of the men is ever next to his own wife?”. It is the topic of my paper on Undermining Meaning in *Journal of Macrodynamical Analysis* (1) 2001 [<http://www.mun.ca/jmda/vol1>]. There I offer you a decent beginning. The objective is to reach a theoretic control of meaning, and a control of that meaning, and eventually - think of the third order of consciousness - a control of your biography of control or its lack. You will know that you have reach a theoretic control when you are able to teach, without notes, the answer. Only you can decide effectively whether this is worth a month of your life.

¹⁴²*Insight*,542[566].

procedure.”¹⁴³ Immediately we have the key challenge of this book: the problem of following up the directions, here obviously my directing of you to those thirty pages of **Penrose**. Do you, per chance, have the book? A photo-copy of the section would cost less than \$2.00 (isn't 16 sheets within some legal limit?). Here I can only throw in cautionary and pedagogic nudges. Notice, for starters, that “imagine” places us in the realm of concrete fantasy - a foundational task of minding. The Turing machine takes quite an effort of fantasy: think of it as a training ground for the fantasy required to reach a serious grasp of global functional collaboration.

Here definable means something like the step by step process of getting square roots, but that machine would be quite complex compared to what Penrose asks us to think out initially. He is asking us to understand, with a practical turn of minding, a shockingly simple tape-moving machine. “In Turing’s picture the ‘tape’ consists of a liner sequence of squares, which is taken to be infinite in both directions. Each square on the tape is either blank or contains a single mark. The use of marked or unmarked squares illustrates that we are allowing our ‘environment’ (i.e the tape) to be broken down and described in terms of *discrete* (as opposed to continuous) elements. This seems to be a reasonable thing to do if we wish our device to functional in a reliable and absolutely definite way. We have, however, allowed this ‘environment’ to be (potentially) infinite, as a feature of the mathematical idealization that we using, but in any *particular* case the input, calculation and output must always be *finite*. Thus, although the tape is taken to be infinitely long, there must only be a finite number of actual marks on it. Beyond a certain point in each direction the tape must be entirely blank.”¹⁴⁴

This may put you in mind of computers or generally of binary devices, which minding helps you along. Penrose uses zeros and ones and is generous with sketches and diagrams of 0s and 1s in sequence, and he is also generous in illustrating the details of the simplest of Turing machines. So, one can envisage a tape that ‘says’ 4, to which we need to add one to get it to ‘say’

¹⁴³**Penrose**, 35.

¹⁴⁴**Penrose**, 37.

5.¹⁴⁵ So we need the device to get from ...000011110000... to ...0000111110000... Penrose nudges us towards exercises:

“As an exercise, some dedicated readers might perhaps care to verify that the following explicit description of a Turing Machine (which I shall call **EUC**) does indeed effect Euclid’s algorithm when applied to a pair of unary numbers”¹⁴⁶ and he gives the full details of the back-and-forth of the machine - to the right or left, in large and small steps. There are 22 such defined steps. Quite an exercise if you wish to get into it. But Penrose encourages a more modest effort. “Before embarking on this, however, it would be wise for any such reader to start with something much simpler, such as the Turing machine **UN+ I**, which simply adds one.”¹⁴⁷ That machine is a four-step machine, and he describes the back-and-forth operation that gets us from 4 to 5. The tape halts when the job is done. **Halting**, as we shall see, is going to be an expression of the key problem. We had a simple instance of it in getting the square root: the machine gives a zero, if there is a square root.

The binary coding system can be complexified even to the extent of coping with decimals: again, the computer world makes you sympathetic to this possibility. But you should also be thinking of the Goedel strategy that we touched on: numbering various letters and symbols so as to arrive at a mapping on numbers that can be written out fully in terms of zeros and ones. You need to stretch your imagination all along here. We are talking and thinking of -

¹⁴⁵**Penrose**, 41. Take note and think about the word ‘say’? What do we mean here? Notice - and this is a major feature in our work - that to be luminous about all this, what we and the machines are doing, we have to lift the effort to an understanding of understanding. Does this not throw humbling light on that slogan of *Insight*:” thoroughly understand with it is to understand”? Does it not also invite us to thoroughly understand what it is to thoroughly understand? How thorough need we be, granting that we wish to reach a significant appreciation of the evolutionary throw-up that we are? We are in the thin air of the intelligible in the profounder sense (*Insight*, chapter 19, section 6) We get a nudge in that direction from Penrose in the text, p. 42: “To understand fully why an algorithmic procedure does what it is supposed to involves insights. (italics his). Are ‘insights’ themselves algorithmic? This is a question that will have importance later”. See further note 132 above.

¹⁴⁶**Penrose**, 41.

¹⁴⁷*Ibid.*

how does one think of such bigness, which makes the age of the cosmos seem small? - very big numbers. Later Penrose works out a process of listing various Turing machines, and writes out the number for one of them. It is over half a page long in decimal notation,¹⁴⁸ and more that a page and a half long in binary notation.¹⁴⁹

The one machine he thus numbers is a more complex machine in that it is set up to mimic other machines by being, so to speak, pre-programmed to handle stuff just like the other machine. How is this done? “The basic idea is to code the list of instructions for an arbitrary Turing machine into a string of 0s and 1s that can be represented on a tape. This tape is then used as the initial part of the input for some particular Turing machine U - called a universal Turing machine - which then acts on the remainder of the input just as T would have done. The universal Turing machine is a universal mimic.”¹⁵⁰ Four pages later in **Penrose**, having worked out a suitable system for numbering Turing machines in a list - you recall our solution with the fractions and our problem with the decimals? - we are ready, if we have not got lost in those pages, to get a beginner’s idea of the process. T_n is now the n^{th} in a sound list of machines and there is a “tape” for each such machine. The mimic machine can do its thing, so to speak, on an m tape. Why not let Penrose entertain us with the next packed paragraph: it may remind you of reading *Scientific American* - **entertaining** in a very peculiar sense.¹⁵¹

“What the Turing machine U would have to do, at each successive step of the operation T_n on m, would be to examine the structure of the succession of digits in the expression for n so that the appropriate replacement in the digits for m (i.e. T_n ‘s ‘tape’) can be made. In fact it is not difficult in principle (though decidedly tedious in practice) to see how one might actually construct

¹⁴⁸**Penrose**, 56-57.

¹⁴⁹**Penrose**, 71-73.

¹⁵⁰**Penrose**, 51.

¹⁵¹A precise pedagogy, non-*haute non-vulgarization*, is needed regarding pedagogy and popularization. It must needs be a rather lengthy treatise and it has to include an explanatory account of that peculiar differentiation of consciousness, present performatively both in the mature Zen master and in the good teacher of, say, physics, that has shed illusions of speedy communication.

such a machine. Its own list of instructions would simply be providing a means of re-reading the appropriate entry in the ‘list’ which is encoded in the number n , at each stage of application to the digits on the ‘tape’, as given by m . There would admittedly be a lot of dodging backwards and forwards between the digits of m and those of n , and the procedure would tend to be exceedingly slow. Nevertheless, a list of instructions for such a machine can certainly be provided: and we call such a machine a *universal* Turing machine. Denoting the action of this machine on the pair of numbers n and m by $U(n,m)$, we have:

$$U(n,m) = T_n(m)$$

for each (n,m) for which T is a correctly specified Turing machine. The machine U , when first fed with the number n , precisely imitates the n th Turing machine.”¹⁵²

Without the back-up of slow work on **Penrose** I doubt if this paragraph makes much sense. How much sense?: that is a topic of the next section. But perhaps you have the impression that somehow Turing is heading in the same direction as Goedel. Indeed, that is the purpose of the massive exercise: to find out whether there is some mechanical procedure for answering all mathematical problems. Turing puts it in terms of the problem of whether or not the n th Turing machine would actually stop when acting on the number m . This is what is referred to as *the halting problem*. Let us say that we expressed the non-stopping as $T_n(m) = !$ ¹⁵³

Patience: another step and we will bring it all oddly together in section 8: or should I say section 9?. We? Bring together? Well, these are the tricky questions of section 9!

There follows here in **Penrose** another 8 pages of pretty dense stuff, including a distraction on Fermat’s theorem which we follow up in section 9. Can I bridge this gap by giving some gist ..? And is the answer to this question on your part a luminous **no?**

Well, suppose we had a machine that would do the trick. That machine works thus: it comes out 0 if $T_n(m) = !$; it comes out 1 if $T_n(m)$ stops. If $T_n(m)$ stops it gives some value, say

¹⁵²**Penrose**, 56.

¹⁵³**Penrose** does not use an exclamation mark, but a square here. I use the exclamation mark because I cannot type a square: but the exclamation mark does capture the surprise element of these theorems. The Hilbert Program is blocked!

p: in symbols we can say $T_n(m) = p$.¹⁵⁴ Now think of the full listing of this process: and with n and m there is the hint of a display, which should remind you of the strategy we used to have a shot at listing the decimals. So, let n number the lines downwards, and let m number the columns across.

m >	0	1	2	3	4	5	6	7	8
n										
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1
3	0	2	0	2	0	2	0	2	0
.										
7	0	1	2	3	4	5	6	7	8
.										
.										
.										
197	2	3	5	7					

If you are working with **Penrose** you notice that I am skipping steps, and the skipping stops the gist being anything like a gist. But I wanted, in our sketching, to get us to the point of seeing a parallel between Turing’s work and Cantor’s: “We now apply a variant of an ingenious and powerful device, the ‘diagonal slash’ of Cantor.”¹⁵⁵ So, we go down the diagonal, adding 1 to each number. That gives us - we skip along merrily - a computable number that is not on the list. But we supposed we had a full listing!

You have some sense of a parallel with Cantor here? How about the parallel with Goedel

¹⁵⁴I am skipping along here; what else can I do? Find out the problem for yourself by understanding Penrose sufficiently well to attempt a presentation to some audience. The audience for a brief presentation is in the future. I note that you can do the same exercise with regard to the problem that I posed in note 152. It is a wonderful exercise in the control of meaning, carrying forward a sufficiently sophisticated audience without either you leaning on notes or they leaning on illusions of comprehension. In my own experience it can be a thrilling two hours.

¹⁵⁵**Penrose**, 62.

as we left him in section X above? At least you can notice that the diagonalizing process gives something like the ‘self-reference’ that seems to come with Goedel’s formula: down the diagonal we have $T_n(n)$. But there are a fair number of twists and turns in the next three pages of **Penrose** before he is satisfied that the contradiction has been expressed; the list is incomplete: this is revealed to him through thinking through the effect of finding a k for which $T_k(k)$ can neither stop nor not stop. I do not wish to get us into that, but I would like us to pause over remarks on this final page 65 of the Turing section. “.... Thus $T_k(k)$ cannot stop i.e $T(k) = !$ But the algorithm cannot ‘know’ this” There is a way in which we can use the algorithm to improve the situation, since “in effect, it ‘knows’ that $T_k(k) = !$ - or does it? It has been helpful in the above description to use the anthropomorphic term ‘know’ in reference to an algorithm. However, is it not we who are doing the ‘knowing’, while the algorithm just follows the rules we have told it to follow? Or are we ourselves merely following the rules that we have been programmed to follow from the construction of our brains and from our environment? The issue is not really simply one of algorithms, but also a question of how one judges what is true and what is not true. These are the issues that we shall return to later.”¹⁵⁶

We turn to that central question of truth here in chapter 2, sketchily, doctrinally. The issue in this present chapter is an issue of identification, self-identification, a self-identification made possible by the searchings of Goedel, Turing, Penrose, etc. The self-identification is to lead to a revision of The Standard Model, slowly, perhaps only in a hundred years or so. But puttering with the self-identification, as we are doing here, can could, should, lead us to a self-identification that has been missed in the reading of *Insight* for the past fifty years. But before we get into that we need to return to Goedel for some further musings.

8. Goedel Again

We can pick up where we left off with J.L.Lucas. “The essence of the Goedel formula is that it is self-referring. It says that ‘the formula is unprovable-in-the-system’.¹⁵⁷ We have heard

¹⁵⁶**Penrose**, 65.

¹⁵⁷See note 45 above for the fuller text and the reference.

something similar from Penrose in the quotations just above. For our reflections here we had best stick with the relevant formula as we met it earlier:

$$G. \quad (x) - \text{Dem} (x, \text{sub} (n, 13, n)).$$

We got this formula, you can recall, by starting with a less twisting formula,

$$A. \quad (x) - \text{Dem} (x, \text{sub} (y, 13, y)).$$

The expression $\text{sub} (y, 13, y)$ designates a number, the number being the Goedel number of the formula got from the formula with Goedel number y by substitution for the variable with Goedel number 13 the numeral for y .¹⁵⁸ This formula has a Goedel number: we call it n . Then we get to G by substituting for the variable with Goedel number 13 - which is y - in the formula A , the numeral for n .

We take now for our contemplation - and I mean that in a very full sense - two paragraphs of Nagel and Newman.¹⁵⁹

“This formula, G , occurs within the arithmetical calculus, and therefore must have a Goedel number. What is that number? A little reflection shows that it is $\text{sub} (n, 13, n)$. To grasp this, we must recall that $\text{sub} (n, 13, n)$ is the Goedel number of the formula that is obtained from the formula with Goedel number n by substituting for the variable with Goedel number 13 (i.e. for the variable ‘ y ’) the numeral for n . But the formula G has been obtained from the formula with Goedel number n (i.e from the formula displayed as A) by substituting for the variable ‘ y ’ occurring in it the numeral for n . Hence the Goedel number of G is in fact $\text{sub} (n, 13, n)$.

But we must also remember that the formula G is the mirror image within the arithmetic calculus of the meta-mathematical statement: ‘The formula with Goedel number $\text{sub} (n, 13, n)$ is not demonstrable’. It follows that the arithmetical formula ‘ $(x) - \text{Dem} (x, \text{sub} (n, 13, n)$ represents in the calculus the metamathematical statement: ‘The formula ‘ $(x) - \text{Dem} (x, \text{sub} (n, 13, n)$ ’ is not demonstrable. In a sense, there fore, this arithmetical formula G can be construed as asserting of itself that it is not demonstrable”.

¹⁵⁸**Goedel’s Proof** backs up such talk with four or five pages of footnotes. This puts us in the same position as we were in with **Penrose**. However, I must assume that if you are seriously interested in this business you will pick up a copy of this paperback.

¹⁵⁹**Goedel’s Proof**, 89-90.

Are you up to this challenge? I would, of course, like to have feedback on this. The equivalent of these two paragraphs were my stumbling block for those months in the 1960s.¹⁶⁰ I think that the problem of handing them luminously troubled Lonergan, and that this was at the centre of his late questioning. His answer to the question regarding the significance of Goedel, as I mentioned already, turned towards the goal of human inquiry; my invitation here is for us to reflect on the lower ground, the empirical context, of our inquiry. Might you stay with me, with this, for an hour, for a month? The cosmos brought forth those two paragraphs through 13.7 billion years of the dynamics of the empirical residue, an energy of indeterminate dispersedness, seeking this little embrace,¹⁶¹ this lift by you into a tiny unity of minding.

What you are minding is dispersed: two physico-chemical paragraphs on paper or the equivalent in computerland, but eyed by you within. Within? “There exist certain elementary and familiar experiences of looking, moving about, grasping, etc.”¹⁶² But, my my, how they have changed psychically if you have come back with me, after living it up positionally in *Insight*, to a “riverrun past Eve and Adam”¹⁶³ of chapter one of *Insight*, where we are now paused. Then, or THEN, or Ven, if you have stayed with it, within, the meant of the two paragraphs, the two paragraphs - but spread in sensability’s duration and extension, are in the surround of your wonder-skin with its capturing context, perhaps that context even holding and held in The Standard Model?¹⁶⁴ Are we with Augustine, or reaching to be with Augustine, or reaching beyond Augustine, in the third mode of generalized empirical method? But our reach has been lifted, is

¹⁶⁰We have not arrived here at the Goedelian result, but it is a turning point that merits our attention. One carries on from there without much mental sweat, in whatever version you are working.

¹⁶¹Recall the various occurrences of “embrace” in *Insight* e.g. 417[442] One may even reach for a *Completion (Method, 250)* of that recall in adverting to the Embrace that is the third Person of the Trinity, remembering that that Embrace seeks the little embrace but in a wonderful tri-personal Oneness.

¹⁶²*Insight*, 142[166].

¹⁶³Joyce, *Finnegans Wake*, first line.

¹⁶⁴*Verbum*, 238.

being lifted, by shifts in the culture of expression, be it in music or mathematics, to critical subtleties of reference and content-precision. A century separates us from the full refinements of linguistic-feedback of The Standard Model of 2100. But we can suspect that the loose controlling insights of such as Goedelian formulae or Turing machines will corner the oversights that cloud our empirical searchings at present. The formula G or its equivalents does not speak for itself, cannot be “construed as asserting of itself” anything, and algorithms “do not know” how neatly they work or halt.

But one must start, alone and without linguistic feedback or suitable controlling expression, to ingest just what is going on when one reads a line such as

(x) -Dem (x, sub (n, 13, n) has Goedel number sub (n, 13, n).

What is one’s - your - interior word, product of sweating over something like Goedel’s writing up of his creative struggle, of the first part of that line? Are you in control of its double meaning, as well as its lack of self-reference? Self-reference is, literally an achievement of the Gods, and even when you and I say *I*, is the “I” a self-referring or is it not a pointer towards an inner word which is only intentionally **I**?¹⁶⁵ At all events, if one asks what does the first half of the line mean, one asks of oneself what meaning one has for that spread of ink, while one is all the while reaching for Goedel’s or Penrose’s meaning, and that only to reach quite beyond them is their sad truncatedness. The road forwards to a luminous community of metalogicians is a massively strenuous climb round the spiral of The Standard Model.¹⁶⁶

But might I be cheeky, like Fermat, and claim that I have the neatest little expression of an adequate halting, at a convenient plateau in finality’s exigence, of the problem of thinking that single line? Might it occupy two paragraphs? But would they be anything more than vertical finality’s obscurity, “vertical finality heightened, gently and quietly,”¹⁶⁷ a theorem marking a

¹⁶⁵If one admits angels into one’s cosmic view, there is some tricky thinking to be done about the **I** or **me** of the angel’s knowledge of self. See Aquinas, *SummaTheologiae I*, q.54, aa.1,2.

¹⁶⁶Some few hints about Logic’s future struggle are given in chapter 5 below.

¹⁶⁷I quote from the end of an essay of Lonergan (“Mission and Spirit”, *A Third Collection*) that I would appeal to here as a context, raising the question, “what would be going all the way?”

future community of those who live in The Standard Model?

9. Fermat and Lonergeran

Penrose turns occasionally to Fermat's Last Theorem as an illustration of the problem of halting.¹⁶⁸ The theorem is one that can be put into a Turing-machine format. This can be grasped by puzzling over a suitable expression of it. The normal expression is Fermat's conviction that there are no natural numbers for x , y , z , and w such that $x^w + y^w = z^w$, with w greater than 2. In a format that both suits the Turing analysis and reminds us of the Goedel formula G , we may say that

$$-(w, x, y, z)[(x + 1)^{w+3} + (y + 1)^{w+3} + (z + 1)^{w+3}]^{169}$$

Since Penrose wrote the book, Andrew Wiles has solved the problem.¹⁷⁰ Fermat (1601-1665) left a scribble behind on the margin of Diophantus's *Arithmetica*, that he had "a truly wonderful proof" of the truth of the formula, but lacked the space to write it down. Now, I wish you to enter fantasyland. Lonergeran scribble "a truly wonderful full-proof" page¹⁷¹ on the core of The Standard Model three hundred years after the death of Fermat. Did he spell it out Wiles-style? I would say, no: and what he eventually wrote in the entire book named *Method in Theology* is more like Amir D. Aczel, *Fermat's Last Theorem. Unlocking the Secret of an Ancient Mathematical Problem*.¹⁷² Such a fantasy, if legitimate, leaves us with a massive problem of self-identification in the next century. I have been trying to spell it out since 1969, when I associated

(*Ibid.*, 27).

¹⁶⁸Penrose, 58, 102-3.

¹⁶⁹See Penrose 103 for details.

¹⁷⁰The final collaboratively-produced version is in *Annals of Mathematics*, May 1995, volume 142, 445-551.

¹⁷¹I am thinking here both of the "discovery page" of February 1965 and of that brilliant page 250 of *Method* to which I devoted some hundreds of Wiles pages, to which history is to devote hundreds of years.

¹⁷²Four Walls Eight Windows Press, New York, 1996.

Loneragan's discovery-page with the mess of contemporary musicology.¹⁷³ I do not wish to pursue that problem here. It was the main topic in *Method in Theology and Botany*. It is the other problem of self-identification that predominantly concerns me here and now and right through this book, the problem of the book *Insight*. I put it in an odd but rememberable way by posing the question, Is *Insight* Fermatish? Yes, that represents the extreme suggestion that some substantial amount of the book is like marginal pointing. There was, if I may pun horribly, a lot of Wiley work in the 28 years of Lonergan's climb to the book. And indeed the very writing of it was done in somewhat the same mood of privacy that dominated Wiley's effort.¹⁷⁴ We return to this question in the final chapter here, chapter 14. But I would ask you to relate it to the first chapter of *Insight* and the set of powerful theorems lurking in it.¹⁷⁵ For an elementary nudge you might now re-read the single line: "a second example of inverse insight is the non-countable multitude."¹⁷⁶ We are, in our ramblings here, placed freshly in the world of Cantor's diagonal. But in what sense was it, or is it even now, an example? There is, in fact, a chapter or a lecture in that line of *Insight*: nor did we more that skim past that lecture in what was written above.¹⁷⁷ And

¹⁷³The spelling-out was presented at the International Lonergan Florida Conference of 1970 in a paper entitled "Meta-Music and Self-Meaning". The paper is available on the website www.philipmcshane.ca As chapter 2 of *The Shaping of the Foundations*.

¹⁷⁴The story of Wiley's intense secretiveness is told in the work cited in note 184 above. In the final stages of his struggle, he had to reach out. "He finally decided he could probably not do very well by keeping secrecy forever. As he himself said, one could work on a problem for an entire lifetime and not see any results. The need to compare notes with another person outweighed that intense need to keep it all to himself. But now the question was: who?" (*Op.cit.*, 122-3). Lonergan's solitude was a different matter (See William Mathews, *Lonergan's Quest. A study of Desire in the Authoring of Insight*, University of Toronto Press. 2006), *passim*. I like to draw attention to the fact that in the final ten pages of *Insight*, the word *collaboration* occurs 29 times. It is the core of cosmopolis, of both micro-autonomy and globality, of inter-religious dialogue, of the trinitarian echo in finitude.

¹⁷⁵In chapter 5 of *A Brief History of Tongue* I reflected on aspects of the first chapter of *Insight* in a manner that helps here.

¹⁷⁶*Insight*, 21[46].

¹⁷⁷At this late stage in the chapter you surely have become more aware of the gap between doctrinal writing and pedagogical writing? I refer to this chapter, of course, obviously full of gaps

what of the remark “every schoolboy knows”?¹⁷⁸ What should every foundations girl know about the definition of the circle? In the year in which *Insight* was published Lonergan pointed towards what he had dealt with years previously. “Insight is an elusive thing. You get hold of insights properly only by considering the history of science, the history of philosophy, and so on.... You get insights together insofar as you say, ‘Well, a geometer understands the whole of Euclid, he can tell you where the key propositions are, and prove all the propositions that follow from a given set of axioms. He’s got the whole thing right in his intellectual paw, so to speak.’ But that comprehensive grasp of the whole subject is not some phenomenon that you can pin right down and describe the structure. When you are seeking insight into insight, not only have you a different term of attention, but your methods of procedure have to differ if you are going to get anywhere.”¹⁷⁹

If you are going to get anywhere as a foundations person, your methods of procedure in ingesting chapter one of *Insight* have to break radically with old habits of reading philosophy. Just how radical that break is to be is a matter of fantasy, fantasy to be sustained by the cycling of The Standard Model in this next century. It seems best to leave our musings on that reaching till we pass through the sketchings of these next twelve chapters.

But, back now to that cheeky remark at the end of the last section about ‘Fermating’ that single boldfaced line near the end of section 8. Let us, however, not reach forward into a Wiley way be shift along more elementarily, indeed get back with a twist to Cantor as we met him in the first section. So now the single line I would have you attend to is, say, the single line that begins a proof through diagonalization Take my own silly effort to get out of the problem by beginning with the following line:

and pointings e.g. to the work to be done. So, obviously doctrinal. Chapter 1 of *Insight*, indeed the entire book, is less obvious. In the third stage of meaning the difference between map-reading meaning and climbing towards meaning will be discontinuously more luminous. The high point in personal luminosity is to intussuscept that the elder, or even older, is - normatively - moving at a faster pace than the younger, that this is an inevitability of human life.

¹⁷⁸*Insight*, 7[31].

¹⁷⁹PL, 357.

upsettling that echos Zeno's upsetting race.¹⁸⁵ So we arrive at a new version of *Insight's* ABC, upsetting *Homo Viator*,¹⁸⁶ a Tri-via¹⁸⁷ pursuit that is not trivial but heart holding, if I let it,¹⁸⁸ the exigence that is me.¹⁸⁹ What, then, is staring me in the face?

I - meaning *you* - can go back to mark, or go on here to write out, a decimal which I may call *A*, the first in the list of the proof. Then there is the decimal *B*, the diagonal modified or not. What might I pick as *C*? Perhaps the vertical edge to the right on the listing, down the dotted dotted line, filled in by me if I need the symmetric image - but the dots may suffice for the sophisticate¹⁹⁰ - a third decimal. The three decimals are then laid out, beseeching us, in their

¹⁸⁵See, for example, his ongoing struggle with the continuum problem in Paul Benacerraf and Hilary Putnam, eds, *Philosophy of Mathematics. Selected Readings*, 1964, 1983. Hao Wang gives a sweep through his life and his interests: *Reflections on Kurt Godel*, MIT Press, 1988, deals with this central problem on pp. 2-4-5, 291-5. A worthwhile aside here regards Hao Wang's own philosophic reflections in his *Beyond Analytic Philosophy*, MIT Press, 1988. The fifth chapter is relevant to our reachings here, but I would draw attention especially to the single page 38 where he suggests a **phenomenography**, a "quest for a structured comprehensiveness". I would relate this to my own suggestions regarding a transposition of Derrida's Grammatology (see *Lack in The Beingstalk*, section 5.3). There is needed a massive shift in empiricity in the study of linguistic expression, its simple suggestiveness and its subtle limitations.

¹⁸⁶I would add here, herenow, paging you, the context of the Epilogue of *Wealth of Self and Wealth of Nations*, "Being and Loneliness", where I look to a book that would sublimate the various searchings, *Being and Having*, *Begin and Doing*, *Being and Nothingness*, *Being and Time*. It would sublimate our thrownness, the thronedness, of our there-being.

¹⁸⁷A fresh twist on the Latin root that relates to crossroads talk: the tri-via is the bend in the road of metaphysics that is generalized empirical method in its third mode.

¹⁸⁸Part of the letting is letting oneself be identified by the demands of history, of emergent cyclic collaboration, of global community, of a Cosmopolis identifying itself, of a "Satisfactory Suffering" (the title of *Joistings* 8, a relevant reflection in the present context). "Each of you brings a psalm or some instruction or a revelation, or speaks in a tongue or gives an interpretation. Let all these things be done in a way that will build up the community." (I Cor, 14: 26).

¹⁸⁹See **PL**, 174, 242. Lonergan massively transposes Marcel's reflections on exigence. See the index of **PL** under *Exigence*.

¹⁹⁰Chapter 3 below calls attention to the subtleties to be reached through self-digestion in our use of symbols, making marks mean what we please. But, obviously, it is just a call to

energy-finality, to identify¹⁹¹ their dispersedness - and the dispersedness of our very “T”s¹⁹² - in our thinking of our “existential gap that consists in the fact that the reality of the subject lies beyond our horizon”¹⁹³ in the mysterious field. So we may reach a new frontier of Lonergan’s late question to me about the significance of Goedel’s incompleteness searchings and of his 1957 question about “Enlarging the Significance of the Existential Gap.”¹⁹⁴ In a hundred years or so

attention, a call to be heard in this century of Lonergan studies. We return to that call in the final chapter here, chapter 14, but perhaps you can feel the sense of reaching forward haunting this final notes of the present chapter.

¹⁹¹Another long chapter would be required to share existentially the relevant exercises of identification here. There is a sense in which this is the core exercise of this chapter and the next, caught slightly in the image here of nine nines.

$$\begin{array}{cccc} & & 9 & \\ & & 9 & 9 \\ & 9 & & 9 \\ 9 & 9 & 9 & 9 \end{array}$$

Perhaps the broad question, Could you do mathematics without dispersedness?, helps. You thus merge question of the ABC of *Insight* with problems of *Insight* chapter 10 and *Verbum*’s **intelligible matter**, but now in the new context of generalized empirical method’s third meaning, with the meaning of the skinfilling notion of being that is the incarnate subject slimly self-appreciating, something that Lonergan is reaching for when he writes, talks, in *Phenomenology and Logic* about the subject-as-subject but I have to qualify his **as** so that it is, you could say, a **not-as**, a presence, a cosmic call, a cos - mi - c - all, cosmic call, identified in its dispersedness. The end of chapter 2 of *Lack in the Beingstalk* could help here: how is the room filled with “The Music of the Spheres”? So you may move, self-tastingly, to an incarnate metaphysics, an integrally-orientated subject in luminous darkness, the **character** of *Magna Moralia* [first paragraph], of *Method* ch. 14, section 1, transposed. The foundational person reaches towards heartily envisaging the virtualities of creativity in dispersedness all the Way to the Eschaton. On virtualities, recall note 26 above.

¹⁹²I risk adding here a further strange exercise that has the flavour of Origin’s openness in the face of scripture, mixed with the mood of John Chrysostum when he preaches of Christ saving the air by being lifted up (*De Cruce et Latrone*, hom. 1,2). So, one has a strange shift of meaning for John 12:32 when one thinks of “T” being lifted up, print lifted to inner word with its contextual hidden agenda of nostalgia.

¹⁹³**PL**, 281.

¹⁹⁴The title of section 1.1 of chapter 14 of **PL**.

there could “come about”¹⁹⁵ a more fundamental standard grasp within the Tower of Able of how our horizon is “earthed”, so that the field is more luminously a luminous darkness of circumstances, so that we darkly self-tune to the dynamics of our reach for a glimpse of the lilies of the fields, of the **field**.

¹⁹⁵Does it not seem appropriate to end the footnotes here by noting, with *Insight* 514[537] that crowning organic madness of the existential subjects of the Tower, no longer bewildered (*Insight*, 385[410]) but spiraling, coming about ever-freshly, in the clear darkness of Cosmopolis?